



UNIVERSITY OF SARGODHA
OFFICE OF THE REGISTRAR
(ACAD BRANCH)

NOTIFICATION

On the recommendations of Academic Council made in its 24th (1/2025) meeting held on 26.08.2025, the Syndicate in its 72nd (4/2025) meeting held on 12.09.2025 has approved the revised curricula of following programs for implementation w.e.f. Fall 2025.

- | | | |
|------|---|-------------|
| I. | Associate Degree in Mathematics | (Annex-'A') |
| II. | BS in Mathematics | (Annex-'B') |
| III. | BS in Mathematics (5 th Semester Intake) | (Annex-'C') |


(WAQAR AHMAD)

Additional Registrar (General)

Dated: 03.11.2025

No. SU/Acad/25/ 1184

Distribution:

- Chairman, Department of Mathematics
- Controller of Examinations
- Director Academics

C.C:

- Dean Faculty of Sciences
- Director, QEC
- Additional Registrar (A & R) *{With the request to forward the notification alongwith curriculum to all Principals of affiliated colleges concerned}*
- Secretary to the Vice-Chancellor
- PA to Registrar
- Notification File

SCHEME OF STUDIES

BS Mathematics (5th Semester Intake)
(w.e.f. FALL-2025)



DEPARTMENT OF MATHEMATICS
UNIVERSITY OF SARGODHA
SARGODHA

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1. Title of Degree Program: BS Mathematics (5th Semester Intake)
2. Program Learning Objectives:

Graduates will be able to

- understand the mathematics theoretically and then visualized by computer programming.
- utilize the knowledge for professional skill development.
- demonstrate good communication skills in professional and academic presentations.
- upgrade knowledge and skills through professional experience and higher studies.

3. Program Structure:

Minimum Credit Hours	74
Discipline Related Courses / Major	51 credit hours (17 courses)
Interdisciplinary / Allied Courses	15 credit hours (5 courses)
General Education	02 credit hours (2 courses)
Internship	3 credit hours
Capstone	3 credit hours
Program Duration	Minimum: 2 years (4 regular semesters) Maximum: 3 years (6 regular semesters) The maximum limit is further extendable in accordance with HEC semester rules.
For Candidates having Associate Degree or equivalent in same field	Eligibility: At least 45% marks in B.Sc. with Mathematics A & B courses OR At least 2.5 CGPA out of 4.00 in AD in Mathematics or equivalent qualification. Merit Criteria: Total marks obtained in BSc or equivalent qualification out of 800 marks + 20 Marks for Hafiz-e-Quran (if applicable). Note: The conversion formula will be applied in case of semester system (ADS/ADP programs)
For Candidates where disciplines of Associate Degree or equivalent and Undergraduate program are different	At least 45% marks/2.5 CGPA out of 4.00 in ADS/ADP in Statistics, Physics, Computer Science, and Economics or equivalent qualification.
Semester Duration	16-18 weeks for regular semesters (1-2 weeks for examination)
Course Load (per semester)	15-21 credit hours for regular semesters
3 Credit Hours (Theory)	2 classes (1.5 hours each) / 3 classes (1 hour each) per week throughout the semester.
1 Credit Hours (Lab / Field Work)	1 credit hour in laboratory or practical work / project requires lab contact of 3 hours per week throughout the semester.



4. General Education:

Sr. No.	Course Code	Course Title	Credit Hours	Prerequisite
1.	URCG-5129/ URCG-5131	Understanding of Holy Quran-I*/Ethics-I**	1(0-1)	Nil
2.	URCG-5130/ URCG-5132	Understanding of Holy Quran-II**//Ethics-II**	1(0-1)	Nil
Total Credit Hours			02	

5. Single Major Courses:

Sr. No.	Course Code	Course Title	Credit Hours	Prerequisite
1.	MATH-6101	Real Analysis-I	3(3-0)	Nil
2.	MATH-6102	Topology	3(3-0)	Nil
3.	MATH-6103	Partial Differential Equations	3(3-0)	Nil
4.	MATH-6104	Numerical Analysis-I	3(3-0)	Nil
5.	MATH-6105	Real Analysis-II	3(3-0)	MATH-6101
6.	MATH-6106	Complex Analysis	3(3-0)	Nil
7.	MATH-6107	Numerical Analysis-II	3(3-0)	MATH-6104
8.	MATH-6111	Differential Geometry	3(3-0)	Nil
9.	MATH-6114	Integral Equations	3(3-0)	Nil
10.	MATH-61xx	Elective-I*	3(3-0)	Nil
11.	MATH-61xx	Elective-II*	3(3-0)	Nil
12.	MATH-61xx	Elective-III*	3(3-0)	Nil
13.	MATH-61xx	Elective-IV*	3(3-0)	Nil
14.	MATH-61xx	Elective-V*	3(3-0)	Nil
15.	MATH-61xx	Elective-VI*	3(3-0)	Nil
16.	MATH-61xx	Elective-VII*	3(3-0)	Nil
17.	MATH-61xx	Elective-VIII*	3(3-0)	Nil
Major Courses Credit Hours Total			51	

6. Interdisciplinary/Allied courses: 15 credit hours:

1.	CSIT-6101	Programming Languages for Mathematicians	3(2-1)	Nil
2.	PHYS-6130	Introduction to Mechanics	3(2-1)	Nil
3.	AIDC-4102	Machine Learning	3(2-1)	Nil
4.	EDUC-6129	Scientific Writing & Research Methods	3(3-0)	Nil
5.	PHYS-6131	Introduction to Classical Mechanics	3(3-0)	Nil
Interdisciplinary Courses Credit Hours Total			15	

7. Field experience/internship: 03 credit hours:

(Lasting 6-8 weeks and ideally scheduled during summer breaks after 6th semester)

1.	MATH-6112	Field experience/Internship	3(0-3)	Nil
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8. Capstone project: 03 credit hours:

(This project, after the sixth semester, requires faculty supervision and evaluation following department guidelines)

1.	MATH-6113	Capstone project	3(0-3)	Nil
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* These courses are elective & can be selected from the list of elective courses:

Note: These courses will be offered by the department from the list of concentration elective courses as per availability of the resources.

Scheme of Studies
BS Mathematics (5th Semester Intake)

Semester-V

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Major-11	MATH-6101	Real Analysis-I	3(3-0)	Nil
Major-12	MATH-6102	Topology	3(3-0)	Nil
Major-13	MATH-6103	Partial Differential Equations	3(3-0)	Nil
Major-14	MATH-6104	Numerical Analysis-I	3(3-0)	Nil
Indn-3	CSIT-6101	Programming Languages for Mathematicians	3(2-1)	Nil
Indn-4	PHYS-6130	Introduction to Mechanics	3(3-0)	Nil
Deficiency-I	MATH-5105	Abstract Algebra	3(3-0)***	Nil

Semester Total Credit Hours: 18

Semester-VI

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Major-15	MATH-6105	Real Analysis-II	3(3-0)	MATH-6101
Major-16	MATH-6106	Complex Analysis	3(3-0)	Nil
Major-17	MATH-6107	Numerical Analysis-II	3(3-0)	MATH-6104
Major-18	MATH-6108	Mathematical Methods*	3(3-0)	Nil
Indn-5	AIDC-4102	Machine Learning	3(3-0)	Nil
Indn-6	PHYS-6131	Introduction to Classical Mechanics	3(3-0)	Nil
Deficiency-II	MATH-5106	Vector and Tensor Analysis	3(3-0)***	Nil
GE-1	URCG-5129/ URCG-5131	Understanding of Holy Quran-I*/Ethics-I**	1(0-1)	Nil

Semester Total Credit Hours: 19

Semester-VII

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Major-19	MATH-6111	Differential Geometry	3(3-0)	Nil
Major-20	MATH-61xx	Elective-II*	3(3-0)	Nil
Major-21	MATH-61xx	Elective-III*	3(3-0)	Nil
Major-22	MATH-61xx	Elective-IV*	3(3-0)	Nil
Indn-07	EDUC-6129	Scientific Writing & Research Methods	3(3-0)	Nil
Compulsory	MATH-6112	Field experience/Internship	3(0-3)	Nil
Deficiency-V	MATH-5108	Fundamental of Mechanics	3(3-0)***	Nil

Semester Total Credit Hours: 18

Semester-VIII

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Major-23	MATH-6114	Integral Equations	3(3-0)	Nil
Major-24	MATH-6115	Functional Analysis*	3(3-0)	Nil
Major-25	MATH-61xx	Elective-VI*	3(3-0)	Nil
Major-26	MATH-61xx	Elective-VII*	3(3-0)	Nil
Major-27	MATH-61xx	Elective-VIII*	3(3-0)	Nil
Compulsory	MATH-6113	Capstone Project	3(0-3)	Nil
Deficiency-VI	MATH-5109	Number Theory	3(3-0)***	Nil
GE-2	URCG-5130/ URCG-5132	Understanding of Holy Quran-II*/Ethics-II**	1(0-1)	Nil

Semester Total Credit Hours: 19: Total Credit Hours: 74

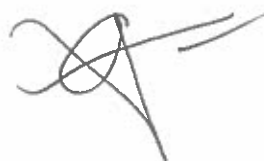


List of Elective Courses

Course Code	Course Title	Credit Hours	Pre-Requisite
MATH-6108	Mathematical Methods	3(3-0)	Nil
MATH-6115	Functional Analysis	3(3-0)	Nil
MATH-6116	Special Functions	3(3-0)	Nil
MATH-6117	Graph Theory	3(3-0)	Nil
MATH-6118	Advanced Group Theory-I	3(3-0)	Nil
MATH-6119	Advanced Group Theory-II	3(3-0)	MATH-6118
MATH-6120	Rings & Modules	3(3-0)	Nil
MATH-6121	Field Theory	3(3-0)	Nil
MATH-6122	Fluid Mechanics-I	3(3-0)	Nil
MATH-6123	Fluid Mechanics-II	3(3-0)	MATH-6122
MATH-6124	Operations Research-I	3(3-0)	Nil
MATH-6125	Operations Research-II	3(3-0)	MATH-6124
MATH-6126	Analytical Dynamics	3(3-0)	Nil
MATH-6127	Special Relativity	3(3-0)	Nil
MATH-6128	Heat Transfer	3(3-0)	Nil
MATH-6129	Measure Theory	3(3-0)	Nil
MATH-6130	Theory of Splines-I	3(3-0)	Nil
MATH-6131	Theory of Splines-II	3(3-0)	MATH-6130
MATH-6132	Methods of Optimization-I	3(3-0)	Nil
MATH-6133	Methods of Optimization-II	3(3-0)	MATH-6132
MATH-6134	Bio-Mathematics	3(3-0)	Nil

**These are deficiency courses. They will be offered during regular Semesters-V, VI, summer semester, VII, VIII and Chairman/Graduate program committee, Department of Mathematics will be authorized to decide the deficiency courses of each student those who got admission after Associate degree other than Mathematics or University of Sargodha and the courses will be offered as per availability of the resources.

***Credit hours are not included for computation of GPA/CGPA. Student will pass this course (if offer)



List of General Education Courses

1-Course Description

The Ethics-1 course is designed to provide students with a comprehensive understanding of ethical principles, practices, and theories in various societal contexts. Throughout this degree program, students will explore the complexities of ethical theories of semitic and non-semitic religions along with decision-making and develop critical thinking skills to navigate moral dilemmas. This course will also enable the students to interact with others religious identities with humanistic, inclusive and holistic approach

2- Learning Objectives

This course aims to:

1. Introduce students to the fundamental concepts, scope, and importance of ethics.
2. Explore the relationship between law, morality, and social values.
3. Develop a clear understanding of virtuous and immoral ethics and their impact on individual and collective life.
4. Study the role of major religious figures in the moral development of human society and enable students to apply ethical principles for personal development, conflict resolution, and social harmony.

3- Learning Outcomes

By the end of the course, students will be able to:

1. Students will be able to identify and analyze major ethical theories, values, and their scope in social and individual life.
2. Differentiate between law and ethics, and analyze their interrelationship.
3. Identify types of virtuous and immoral ethics and assess their social impacts.
4. Examine the ethical teachings of major religions and their relevance in contemporary society.
5. Apply ethical principles to address modern challenges in personal and professional life.

4-Course Structure

1. Interactive lectures, Group discussions and debates
2. Reflection papers and presentations
3. Assignments and Quiz

Course Contents**Unit 1: Introduction and Fundamentals of Ethics**

1. Literal and terminological definition of ethics
2. Literal and terminological definition of values
3. Relationship between law and ethics
4. Need, importance, and scope of ethics

Unit 2: Types of Ethics and Their Impact on Society

- Virtuous ethics: concept, types, benefits, and outcomes
- Immoral ethics: concept, types, and harms
- Role of ethics in social refinement and establishment of peace

Unit 3: Virtuous Ethics (Akhlak-e-Hasanah)

- Concept, need, and importance of virtuous ethics
- Scope of virtuous ethics in the light of religions
- Major virtues in revealed and non-revealed religions
- Impact of virtuous ethics on individual and collective life

Unit 4: Immoral Ethics (Akhlak-e-Ruzifah)

- Concept of immoral ethics
- Social problems caused by immoral ethics
- Practical consequences of immoral ethics
- Major vices in revealed and non-revealed religions

Unit 5: Role of World Religious Figures in Moral Development

- Prophet Moses (AS): Introduction, miracles, and role in moral refinement
- Prophet Jesus (AS): Introduction, miracles, and role in moral refinement
- Prophet Muhammad (ﷺ): Introduction, miracles, and role in moral refinement

Textbook

1. Izutsu, T. (2002). *Ethico-Religious Concepts in the Qur'un*. McGill-Queen's University Press.

Suggested Readings

1. Gert, B. (2005). *Morality: Its Nature and Justification*. Oxford University Press.
2. MacIntyre, A. (2007). *After Virtue: A Study in Moral Theory*. University of Notre Dame Press.
3. Al-Ghazali, Abu Hamid (2001). *The Alchemy of Happiness*. Islamic Texts Society.
4. Nasr, S. H. (1994). *The Heart of Islam: Enduring Values for Humanity*. Harper One.
5. Beauchamp, T. L., & Childress, J. P. (2019). *Principles of Biomedical Ethics*. Oxford University Press.
6. Hasan, Z. (2010). *Ethics in Islam: Key Concepts and Contemporary Challenges*. Islamic Research Institute.

1-Course Description

The course *Ethics-II* is designed to provide students with a deeper understanding of ethical principles and practices from both Semitic and non-Semitic religions, as well as their application in professional and social contexts. Students will engage with reasoning, decision-making, tolerance, and peacebuilding. It aims to cultivate an inclusive, humanistic, and holistic approach towards ethical living and interfaith engagement.

2- Learning Objectives

The course objectives are to:

1. Understand the fundamental principles and theories of ethics.
2. Introduce the ethical and moral teachings of Judaism, Christianity, Islam, and Hinduism.
3. Explore the ethical teachings of non-Semitic religions such as Buddhism, Sikhism, Confucianism, and Jainism.
4. Develop critical thinking skills to evaluate ethical arguments and theories.
5. Promote ethical leadership and interfaith harmony.

3- Learning Outcomes

By the end of this course, students will be able to:

1. Identify and analyze major ethical theories and teachings from world religions.
2. Understand the role of religions in improving moral values and social behavior.
3. Demonstrate ethical decision-making in various personal and professional contexts.
4. Recognize the impact of ethical decisions on individuals, communities, and society.
5. Apply skills of ethical leadership, including communication, conflict resolution, and inclusive engagement.

4-Course Structure

1. Interactive lectures, Group discussions and debates
2. Reflection papers and presentations
3. Assignments and Quiz

Course Title: ~~Ethics-II (For Non-Muslim Students)~~ Course Code: ~~URCG-5132~~ X

Course Contents**Unit 1: Ethical Teachings of Semitic Religions**

- Judaism and its ethical teachings
- Christianity and its ethical teachings
- Islam and its ethical teachings

Unit 2: Ethical Teachings of Non-Semitic Religions

- Hinduism and its ethical teachings
- Sikhism and Buddhism: ethical values and practices
- Confucian and Jain ethical traditions

Unit 3: Professional Ethics

- Ethics for students and teachers
- Ethics in doctor-patient relationships
- Ethics in trader-customer interactions

Unit 4: Concept and Significance of Tolerance

- Definition, need, and importance of tolerance
- Teachings of Semitic religions on tolerance and their contemporary relevance
- Teachings of non-Semitic religions on tolerance and their contemporary relevance

Unit 5: Foundational Values and Ethics for Peacebuilding in Society

- Respect for sacred scriptures, personalities, places of worship, and religious symbols
- Promotion of tolerance and broadmindedness
- Encouragement of dialogue and harmony
- Benevolence towards humanity
- Establishment of justice and fairness
- Patience, forbearance, and forgiveness

Textbook

- Kidder, R. M. (2009). *How Good People Make Tough Choices: Resolving the Dilemmas of Ethical Living*. Harper.

Suggested Readings

1. Barash, D. P., & Webel, C. P. (2014). *Peace and Conflict Studies*. Sage.
2. Smart, N. (1998). *The World's Religions*. Cambridge University Press.
3. Nasr, S. H. (2003). *The Heart of Islam: Enduring Values for Humanity*. HarperOne.
4. Sharma, A. (2006). *Hindu Ethics: Purity, Abortion, and Euthanasia*. SUNY Press.
5. Harvey, P. (2000). *An Introduction to Buddhist Ethics: Foundations, Values and Issues*. Cambridge University Press.
6. Coward, H., & Perkinson, J. (2013). *A Cross-Cultural Dialogue on Ethical Leadership*. Wilfrid Laurier University Press.
7. Confucius. (1998). *The Analects*. Oxford University Press.

Annex - A

URCG-5129

Model Course Outline for the Course Understanding of Quran – I

Course Title: Understanding of Quran – I

Course Book: Muallim ul Quran (Volume 1, 2 & 3) by Dr Ubaid ur Rahman

Credit Hours: 1 (0-1)

Contact Hours: 3 per week

Weeks: 15-16 (45-48 hours)

Course Learning Outcomes:

By the end of this course, students will be able to:

1. Develop the ability to understand basic words of the Quran, phrases and sentences that do not contain verbs (unit 1 to 5 of Muallim ul Quran Book) and then sentences having present tense (first half of unit 6 of Muallim ul Quran Book).
2. Acquire a strong foundation for understanding long verses of the Quran with clarity.
3. Comprehend Quranic vocabulary, particles (operative & non operative particles), compounds (Adjective & Possessive compound), pronouns (singular & plural) and types of plural through hundreds of Quranic sentences.
4. Recognize and understand different styles of Quranic sentences, including nominal sentence, emphatic sentence, double emphatic sentence, negative sentence, interrogative sentence, oath-based sentences.
5. Strengthen understanding of fundamental Quranic linguistic styles, expressions and idioms.
6. Understand at least 30 to 40 % of each page of the holy Quran.

Provision of material, content and books:

- Paper book: All volumes are available in printed book form.
- Tutorial videos: Teaching video of each lesson available on YouTube.
- Confirmation Videos: A complete series of confirmation videos of all lessons is available in which the student can confirm his answers.
- A flipbook: A flipbook edition is also accessible.
- Helping material: Helping material for the teachers like quizzes, question papers and images is available on website.

Course Outline:

Weeks	Lectures (1.5 hrs)	Units	Lessons	Assignments/Home Task	Linguistic Rules
1.	1.	1	1-6	Writing the meaning of Quranic words Lesson 1-8	Proper Noun Masculine & Feminine
	2.	1	9-14	Writing the meaning of Quranic words 9-14	Two kinds of plural Concept of (و) "And" Common Noun
2.	1.	1	15-17	Writing the meaning of Quranic words, phrases & translation of Sentences 15-17	Demonstrative Noun (This & That for Masculine (هذه) Demonstrative Noun (This & That for Feminine) (تلك))
	2.	1	18-19 & Revision (Unit 1)	Writing the meaning of Quranic words, phrases & translation of Sentences 17-19 Quiz	Laam for emphasis (لام التأكيد) أكبر Superlative Degree like أكبر Revision of all Quranic Sentences
3.	1.	Unit 2	1-3	Writing the meaning of Quranic words, phrases & translation of Sentences 1-3	Emphatic Particle (ان) Preposition "For" (اللام) (في) Preposition (في)
	2.	2	4-6	Writing the meaning of Quranic words, phrases & translation of Sentences 4-6	Preposition (على- من- إلى)
4.	1.	2	7-9	Writing the meaning of Quranic words & translation of Sentences 7-9	Preposition (إلى) Absolute Negation Particle (لا) Exceptive Particle (لا التالية) (لما التالية) (للجس)
	2.	2	10-13 & Revision (Unit 2)	Writing the meaning of Quranic words, phrases & translation of Sentences 10-13 Quiz	Subordinating Conjunction (أن), Was (كان), Vocative Particle (حرف النداء)

5.	1.	Unit 3	1-2	Writing the meaning of Quranic phrases 1-2	Quranic Adjective Compounds (صلة وموصوف) (مضاف ومضاف إليه)
	2.	3	3-5	Writing the meaning of Quranic phrases & translation of sentences 3-5	Quranic Possessive Construction (مضاف ومضاف إليه)
6.	1.	3	6-7	Writing the meaning of Quranic phrase translation of sentences 6-7	Quranic Possessive Construction (مضاف ومضاف إليه)
	2.	3	8-10 & Revision (Unit 3)	Writing the meaning of Quranic phrase & translation of sentences 8-10 Quiz	Active Participle (اسم الفاعل), Passive Participle (اسم المفعول), Dual (مثنى)
7.	1.	Unit 4	1-2	Writing the meaning of Quranic phrase & translation of sentences 1-2	Personal Pronoun He (هو) (المتصل) Possessive Pronoun His (له) (المتصل)
	2.	4	3-4	Writing the meaning of Quranic phrase & translation of sentences 3-4	Possessive Pronoun with prepositions like في بيته Pronoun "His" with prepositions like فيه له، منه، فيه
8.	1.	4	5-8	Writing the meaning of Quranic sentences 5-8	Personal Pronoun You (أنت) (المتصل) Possessive Pronoun Your (لك) (المتصل) Possessive Pronoun with prepositions like في بيتك Pronoun "your" with prepositions like فيك لك، ملك، فيك
	2.	Mid-Term			

9.	1.	4	9-12	Writing the meaning of Quranic phrases & sentences 9-12	Personal Pronoun She (هي المنفصل) Possessive Pronoun Her (ها المتصل) Possessive Pronoun with prepositions like لي بيتها Pronoun "Her" with prepositions like لها
	2.	4	13-16	Writing the meaning of Quranic phrases & sentences 13-16	Personal Pronoun I (انا المنفصل) Possessive Pronoun Her (ي المتصل) Possessive Pronoun with prepositions like لي بيتي Pronoun "My" with prepositions like لي
10.	1.	4	17 & Revision Unit 4	Revision of all Quranic sentences of Unit 4 Quiz	Adverb (حال)
	2.	Unit 5	1-2	Writing the meaning of Quranic phrases & sentences 1-2	Masculine Plural جمع المذكر السالم و جمع المذكر السالم المسبوق بحرف الجر
11.	1.	5	3-4	Writing the meaning of Quranic phrases & sentences 3-4	Possessive Construction with Plurals جمع المذكر السالم المسبوق بالإضافة
	2.	5	5-6	Writing the meaning of Quranic phrases, sentences & verses 5-6	Personal Pronoun They (هم المنفصل) Possessive Pronoun Their (هم المتصل)
12.	1.	5	7-8	Writing the meaning of Quranic phrases, sentences & verses 7-8	Possessive Pronoun with prepositions like لي بيتهم Pronoun "Their" with prepositions like لهم
	2.	5	9-11	Writing the meaning of Quranic phrases, sentences & verses 9-11	Personal Pronoun You (انتم المنفصل) Possessive Pronoun Your (كم المتصل) Possessive Pronoun with prepositions

3.	1.	5	12-14	Writing the meaning of Quranic phrases & sentences & verses 12-14	في بيتكم like Pronoun "Your" with prepositions like لكم Personal Pronoun We (نحن المنفصل) Possessive Pronoun Our نا المتصل
	2.	5	15-16	Writing the meaning of Quranic sentences & verses 15-16	Possessive Pronoun with prepositions like في بيتنا Pronoun "Our" with prepositions like لنا
4.	1.	5	17-18	Writing the meaning of Quranic sentences & Verses 17-18	Demonstrative Pronoun These, Those (هؤلاء- أولئك)
	2.	5	19-23	Writing the meaning of Quranic sentences & Verses 19-23	ما / إلا، إن / إلا، إنما ليس، ما (/ ألم، أن، بل، كأن) (إلا، ليس، اليوم، يومئذ، سبحانه، ما بينهما، قل، إذن، بنس، نعم، كلا، ما أدراك، حسب، أعلم ب، مصير، مرجع، ديلا(تمييز)
5.	1.	5	Revision Unit 5	Quiz	
	2.	5	1-3 (till Page 16)	Writing the meaning of Quranic Verbs & Translation of Quranic Sentences & Verses (1-3)	Introduction of Present Tense (فعل مضارع) & Verbal Sentence (جملة فعلية) Present Tense الفعل المضارع صيغة المفرد يعلم
6.	1.	6	3 (From Page 17) & 4-5	Translation of Quranic Sentences & Verses 3-5	Present Tense الفعل المضارع صيغة المفرد يعلم
	2.	6	6	Translation of Quranic Sentences & Verses	Present Tense الفعل المضارع صيغة الجمع يعلمون

Annex - B

URCG-5130

Model Course Outline for the Course Understanding of Quran – II

Course Title: Understanding of Quran – II
Course Book: Muallim ul Quran (Volume 3, 4 & 5) by Dr Ubaid ur Rahman
Credit Hours: 1 (0-1)
Contact Hours: 3 per week
Weeks: 15-16 (45-48 hours)

Course Learning Outcomes:

By the end of this course, students will be able to:

1. Directly comprehend hundreds of Quranic sentences & verses.
2. Understand at least 80 to 85 % of each page of the holy Quran.
3. Understand common verses across different Quranic topics.
4. Achieve proficiency in the basic and advance linguistic aspects of the Arabic language.
5. Understand the difference between Quranic verbs in various forms, such as present, past and imperative.
6. Develop the ability to understand long verses of the holy Quran independently and then comprehend their interpretation.

Provision of material, content and books:

- Paper book: All volumes are available in printed book form.
- Tutorial videos: Teaching video of each lesson available on YouTube.
- Confirmation Videos: A complete series of confirmation videos of all lessons is available in which the student can confirm his answers. ✓
- A flipbook: A flipbook edition is also accessible.
- Helping material: Helping material for the teachers like quizzes, question papers and images is available on website.

Course Outline:

Weeks	Lectures	Units	Lessons	Assignments/Home Task	
1.	1.	6	6	Understanding & Translation of Verses	Present Tense صيغة جمع مذكر غائب مثل يمدون
	2.	6	7-8	Understanding & Translation of Verses	Present Tense صيغة جمع مذكر غائب مثل يمدون
2.	1.	6	9-10	Understanding & Translation of Verses	Present Tense صيغة ملود مذكر مخاطب (تعيد) وجمع مذكر مخاطب (تعيدون)
	2.	6	11-12	Understanding & Translation of Verses	Present Tense صيغة جمع مذكر مخاطب (تعيدون)

3.	1.	6	13	Understanding & Translation of Verses	صيغة المتكلم (أعبد) Present Tense صيغة جمع المتكلم (تعبد)
	2.	6	14-15	Understanding & Translation of Verses	Negative Imperative صيغة المفرد وصيغة الجمع , لا تعبد, لا تعبدوا
4.	1.	6	16-17	Understanding & Translation of Verses	Conditional Sentences & masdar moawal (مصدر مؤول)
	2.	6	18-19	Understanding & Translation of Verses	Laam uttaleel (لام التعليل) & Laam ul jhood(لام الجحود)
5.	1.	6	20-21	Understanding & Translation of Verses	Present with object pronouns & Passive Voice
	2.	6	Revision (Unit 6)	Quiz	
6.	1.	Unit 7	1 (sec 1-3)	Understanding & Translation of Verses	Past Tense صيغة المفرد للثائب
	2.	6	1 (Sec 4-5)	Understanding & Translation of Verses	Past Tense صيغة المفرد للثائب
7.	1.	6	1 (Sec 5-6)	Understanding & Translation of Verses	Past Tense صيغة المفرد للثائب
	2.	6	1 (Sec 7-9)	Understanding & Translation of Verses	Past Tense صيغة المفرد للثائب
8.	1.	7	Revision	Understanding & Translation of Verses QUIZ	Past Tense صيغة المفرد للثائب
	2.			MID-TERM	
9.	1.	7	2 (sec 1-2)	Understanding & Translation of Verses	Past Tense صيغة الجمع للثائب عبدوا
	2.	7	2 (sec 3)	Understanding & Translation of Verses	Past Tense صيغة الجمع للثائب عبدوا
10.	1.	7	2 (sec 4-5)	Understanding & Translation of Verses	Past Tense صيغة الجمع للثائب عبدوا
	2.	7	2 (sec 6-7)	Understanding & Translation of Verses	Past Tense صيغة الجمع للثائب عبدوا
11.	1.	7	3 (sec 1-2)	Understanding & Translation of Verses	Past Tense صيغة الجمع للمتكلم عبدنا

	2.	7	3 (sec 2-3)	Understanding & Translation of Verses	Past Tense صفة الجمع المتكلم عدنا
2.	1.	7	3 (sec 3-4)	Understanding & Translation of Verses	Past Tense صفة الجمع المتكلم عدنا
	2.	7	3 (sec 4-5)	Understanding & Translation of Verses	Past Tense صفة الجمع المتكلم عدنا
3.	1.	7	4 (sec 1-2-3)	Understanding & Translation of Verses	Past Tense صفة الجمع المخاطب عدتم
	2.	7	4 (sec 4-5)	Understanding & Translation of Verses	Past Tense صفة الجمع للمخاطب عدتم
4.	1.	7	5-6	Understanding & Translation of Verses Quiz	Past Tense صفة المتكلم والمخاطب عدت ، عدت
	2.	7	7	Understanding & Translation of Verses	Past Tense صفة المؤنث للفاتب عدت
5.	1.	7	8	Understanding & Translation of Verses	Passive Voice (Past Tense) فعل مجهول للمفرد
	2.	7	9	Understanding & Translation of Verses	Passive Voice (Past Tense) فعل مجهول الجمع
6.	1.	8	1-4	Understanding & Translation of Verses	Imperative Verb for singular فعل الأمر للمفرد
	2.	7	5-8	Understanding & Translation of Verses	Imperative Verb for plural فعل الأمر للجمع

List of Single Major Courses

This is the first part of a two-semester course. The first course covers the following fundamentals of mathematical analysis. Real numbers: Properties and operations of real numbers, including order, completeness, and density. Sequences and series: Convergence, divergence, and properties of sequences and series of real numbers. Limits and continuity: Definition and properties of limits, continuity, and uniform continuity. Differentiation: Definition and properties of derivatives, including rules for differentiation. It develops the fundamental ideas of mathematical analysis and is aimed at developing the student's ability, to write solutions to problems & proofs of theorems that meet rigorous standards based on content, organization & coherence, argument & support, & style & mechanics. The importance in other science are as follows: Physics and engineering; Real analysis is used to model and analyze physical systems, including motion, forces, and energies, in Economics; It is applied in economic modeling, particularly in understanding economic growth, optimization, and dynamics, in Computer science; Real analysis is used in algorithm design, computational complexity, and machine learning.

Contents

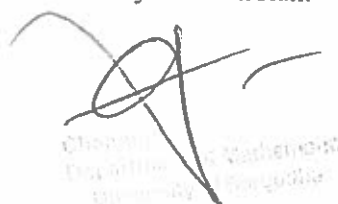
- 1 Number Systems: Ordered fields
- 2 rational, real & complex numbers
- 3 Archimedean property
- 4 supremum, infimum & completeness
- 5 Topology of real numbers
- 6 Convergence, completeness, completion of real numbers
- 7 Heine Borel theorem
- 8 Sequences & Series of Real Numbers
- 9 Limits of sequences, algebra of limits
- 10 Bolzano Weierstrass theorem, Cauchy sequences, \liminf , \limsup
- 11 limits of series, convergences tests, absolute & conditional convergence, power series
- 12 Continuity: Functions, continuity & compactness, existence of minimizers & maximizers
- 13 uniform continuity, continuity & connectedness, intermediate mean value theorem
- 14 monotone functions & discontinuities

Recommended Texts

1. Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4th ed.) New York: John Wiley & Sons.
2. Trench, W. F. (2013). *Introduction to real analysis* (2nd ed.). New Jersey: Prentice Hall.

Suggested Readings

1. Folland, G.B. (1999). *Real analysis* (2nd ed.). New York: John Wiley & Sons.
2. Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.) New York: McGraw-Hill.
3. Royden, H., & Fitzpatrick, P. (2010). *Real analysis* (4th ed.). New Jersey: Pearson Hall.



Chennai
David
Department
of Mathematics
University of
Madras

Topology studies continuity in its broadest context. We begin by analyzing the notion of continuity familiar from calculus, showing that it depends on being able to measure distance in Euclidean space. This leads to the more general notion of a metric space. A brief investigation of metric spaces shows that they do not provide the most suitable context for studying continuity. A deeper analysis of continuity in metric spaces shows that only the open sets matter, which leads to the notion of topological spaces. We easily see that this is the right setting for studying continuity. The central concepts of topology, compactness, connectedness & separation axioms are introduced. Applications of topology to number theory, algebraic geometry, algebra & functional analysis are featured. Since many important applications of topology use metric spaces, we investigate topological concepts applied to them & introduce the notion of completeness. In addition, this course provides the basis for studying differential geometry, functional analysis, classical & quantum mechanics, dynamical systems, algebraic & differential topology.

Contents

- 1 Topological spaces
- 2 Bases & sub-bases
- 3 First & second axiom of countability
- 4 Separability
- 5 Continuous functions & homeomorphism
- 6 Finite product space
- 7 Separation axioms (T_0)
- 8 Separation axioms (T_1)
- 9 Separation axioms (T_2)
- 10 Tychonoff spaces
- 11 Regular spaces
- 12 Completely regular spaces
- 13 Normal spaces
- 14 Product spaces
- 15 Compactness
- 16 Connectedness

Recommended Texts

1. Sheldon, W. D. (2005). *Topology* (1st ed.). New York: McGraw Hill.
2. Willard, S. (2004). *General topology* (1st ed.). New York: Dover Publications.

Suggested Readings

1. Lipschutz, S. (2011). *General topology, Schaum's outline series* (1st ed.). New York: McGraw Hill.
2. Armstrong, M.A. (1979). *Basic topology* (1st ed.). New York: McGraw Hill.
3. Mendelson, B. (2009). *Introduction to topology* (3rd ed.). New York: Dover Publications.



Partial Differential Equations (PDEs) are in the heart of applied mathematics & many other scientific disciplines. The beginning weeks of the course aim to develop enough familiarity & experience with the basic phenomena, approaches, & methods in solving initial/boundary value problems in the contexts of the classical prototype linear PDEs of constant coefficients: the Laplace equation, the wave equation & the heat equation. A variety of tools & methods, such as Fourier series/eigen function expansions, Fourier transforms, energy methods, & maximum principles will be introduced. More importantly, appropriate methods are introduced for the purpose of establishing quantitative as well as qualitative characteristic properties of solutions to each class of equations

Contents

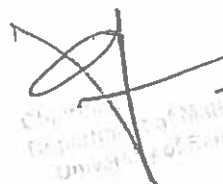
- 1 First order PDEs: Introduction, Formation of PDEs, Solutions of PDEs of first order
- 2 The Cauchy's problem for quasi linear first order PDEs, First order nonlinear equations
- 3 Special types of first order equations Second order PDEs
- 4 Basic concepts & definitions, Mathematical problems, Linear operator
- 5 Superposition, Mathematical models
- 6 The classical equations, The vibrating string, The vibrating membrane
- 7 Conduction of heat solids, Canonical forms & variable
- 8 PDEs of second order in two independent variables with constant & variable coefficients
- 9 Cauchy's problem for second order PDEs in two independent variables
- 10 Methods of separation of variables, Solutions of elliptic
- 11 Laplace transform: Introduction & properties of Laplace transform
- 12 Transforms of elementary functions, Periodic functions, error functions
- 13 Dirac delta function, Inverse Laplace transform, Convolution Theorem
- 14 Solution of PDEs by Laplace transform, Diffusion & wave equations
- 15 Fourier transforms, Fourier integral representation
- 16 Fourier sine & cosine representation, Fourier transform pair
- 17 Transform of elementary functions & Dirac delta function, Finite Fourier transforms
- 18 Solutions of heat, Wave & Laplace equations by Fourier transforms

Recommended Texts

- 1 Myint U. T. Partial Differential Equations for Scientists and Engineers, (3rd ed.). North Holland, Amsterdam, 1987
- 2 Zill, D.G., & Michael, R. (2009). *Differential equations with boundary-value problems* (5th ed.) New York: Brooks/Cole.
- 3 Polking, J., & Boggess, A. (2005). *Differential equations with boundary value problems* (2nd ed.). London: Pearson.

Suggested Readings

- 1 Wloka, J. (1987). *Partial differential equations* (1st ed.). Cambridge: Cambridge University Press.
- 2 Humi, M., & Miller, W. B. (1991). *Boundary value problems & partial differential equations* (1st ed.). Boston: PWS- KENT Publishing Company.


 Department of Mathematics
 University of Singapore

To explore complex systems, physicists, engineers, financiers and mathematicians require computational methods since mathematical models are only rarely solvable algebraically. Numerical methods, based upon sound computational mathematics, are the basic algorithms underpinning computer predictions in modern systems science. The course will cover the classical fundamental topics in numerical methods such as, approximation, numerical linear algebra, solution of nonlinear algebraic systems, matrix decomposition, interpolation and unstable systems. The viewpoint will be modern, with connections made between each topic and a variety of applications. It is optimal to verify numerical methods by using computer programming (Mat Lab, Maple, C++, etc.)

Contents

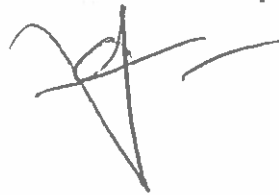
- 1 Error analysis: Floating point arithmetic, Error Approximations
- 2 Methods for the solution of non-linear equations: Bisection method, Regula-Falsi method, Fixed point iteration method, Newton-Raphson method, Secant method, Convergence criteria, Muller's Method, Graeffe's root squaring Method. System of non-linear equations.
- 3 Numerical solutions of a system of linear equations: Direct methods: Gaussian elimination method, Gauss-Jordan method, Pivoting techniques
- 4 LU-decomposition: Crout's method of factorization, matrix inversion, Cholesky's method and Doolittle's method
- 5 Iterative methods: Jacobi method, Gauss-Seidel Method, Relaxation Method. Matrix Norm, Condition Number and Ill-conditioned system
- 6 Eigen values problems: Introduction, Power Method, Jaccobi's Method
- 7 Interpolation: Finite differences operators, Forward differences and Backward difference operators, shift operator, Average operator, Differential operator, Relation between operators, shift operator, Average operator, Newton's forward Differences interpolation formula, Newton's backward differences interpolation formula, Lagrange's interpolation formula, Newton's Divided difference interpolation formula, Interpolation in two dimensions, Hermite interpolation.

Recommended Texts

1. Burden, R. L., Faires, J. D., & Burden, A.M. (2015). *Numerical analysis* (10th ed.). Boston: Cengage Learning.
2. Gerald, C.F., & Wheatley, P.O. (2005). *Applied numerical analysis*. London: Pearson Education, Singapore.

Suggested Readings

1. Philip, J. (2019). *Numerical applied computational programming with case studies* (1st ed.). New York: A press.
2. Khoury, R., & Harder, D.W. (2016). *Numerical methods & modelling for engineering* (1st ed.). London: Springer.
3. Antia, H.M. (2012). *Numerical methods for scientists & engineers* (3rd ed.). New York: Springer.



This course is continuation of Real Analysis I, this course will continue to cover the fundamentals of real analysis, concentrating on the Riemann-Stieltjes integrals, Functions of Bounded Variation, Improper Integrals, convergence of series, study of a selection of theorems concerning integration, to recognize the difference between point wise & uniform convergence of a sequence of functions and to illustrate the effect of uniform convergence on the limit function with respect to continuity. Emphasis would be on proofs of main results. The aim of this course is also to provide an accessible, reasonably paced treatment of the basic concepts & techniques of real analysis for students in these areas. This course provides greatly strengthening student's understanding of the results of calculus & the basis for their validity the uses of deductive reasoning, increasing the student's ability to understand definitions, understand proofs, analyze conjectures, find counter-examples to false statements, construct proofs of true statements & enhancing the student's mathematical communication skills. In addition, it provides the key foundations of related advanced courses e.g. Advanced Calculus, Measure Theory and Functional Analysis.

Contents

1. Differentiation: Mean value theorem, L'Hopital's Rule, Taylor's theorem
2. The Riemann-Stieltjes Integrals
3. Definition & existence of integrals
4. Properties of integrals
5. Fundamental theorem of calculus & its applications
6. Change of variable theorem, integration by parts
7. Functions of Bounded Variation
8. Definition & examples, properties of functions of bounded variation
9. Improper Integrals: Types of improper integrals
10. Tests for convergence of improper integrals
11. Beta & gamma functions
12. Absolute & conditional convergence of improper integrals
13. Sequences & Series of Functions
14. Power series, definition of point wise & uniform convergence
15. Uniform convergence & continuity
16. Uniform convergence & differentiation, examples of uniform convergence

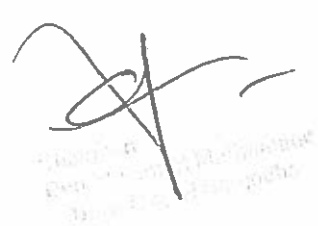
Pre-requisite: Real Analysis-I

Recommended Texts

- 1 Bartle, R. G., & Sherbert, D. R. (2011). *Introduction to real analysis* (4thed.). New York: John Wiley & Sons.
- 2 Rudin, W. (1976). *Principles of mathematical analysis* (3rd ed.). New York: McGraw-Hill.

Suggested Readings

- 1 Folland, G. B. (1999). *Real analysis* (2nd ed.). New York: John Wiley & Sons.
- 2 Hewitt, E., & Stromberg, K. (1965). *Real & abstract analysis*. New York: Springer-Verlag Heidelberg
- 3 Lang, S. (1968). *Analysis I*. Boston: Addison-Wesley Publ. Co.



This is an introductory course in complex analysis, giving the basics of the theory along with applications, with an emphasis on applications of complex analysis & especially conformal mappings. Students should have a background in real analysis (as in the course Real Analysis I), including the ability to write a simple proof in an analysis context. Complex Analysis is a topic that is extremely useful in many applied topics such as numerical analysis, electrical engineering, physics, chaos theory, & much more, & you will see some of these applications throughout the course. In addition, complex analysis is a subject that is, in a sense, very complete. The concept of complex differentiation is much more restrictive than that of real differentiation & as a result the corresponding theory of complex differentiable functions is a particularly nice one.

Contents


- 1 Introduction: The algebra of complex numbers
- 2 Geometric representation of complex numbers
- 3 Polar form of complex numbers
- 4 Powers & roots of complex numbers
- 5 Functions of Complex Variables
- 6 Limit
- 7 Continuity
- 8 Differentiable functions, the Cauchy-Riemann equations
- 9 Analytic functions, entire functions, harmonic functions
- 10 Elementary functions: The exponential, Trigonometric functions
- 11 Hyperbolic, Logarithmic & Inverse elementary functions
- 12 Complex Integrals: Contours & contour integrals, anti-derivatives, independence of path
- 13 Cauchy-Goursat theorem, Cauchy integral formula, Liouville's theorem, Morera's theorem
- 14 Maximum Modulus Principle
- 15 Series: Power series, Radius of convergence & analyticity
- 16 Taylor's & Laurent's series
- 17 Integration & differentiation of power series, isolated singular points
- 18 Cauchy's residue theorem with applications
- 19 Types of singularities & calculus of residues, Zeros & Poles, Mobius transforms
- 20 Conformal mappings & transformations

Recommended Texts

- 1 Mathews J. H., & Howell, R.W. (2006). *Complex analysis for mathematics & engineering* (5th ed.). Burlington: Jones & Bartlett Publication.
- 2 Churchill, R. V., & Brown, J.W. (2013). *Complex variables & applications* (9th ed.). New York: McGraw-Hill.

Suggested Readings

- 1 Remmert, R. (1998). *Theory of complex functions* (1st ed.). New York: Springer-Verlag.
- 2 Rudin, W. (1987). *Real & complex analysis* (3rd ed.). New York: McGraw-Hill.



Numerical analysis is the study of algorithms that use numerical approximation instead of symbolic manipulations for the problems of mathematical analysis. Numerical methods, based upon sound computational mathematics, are the basic algorithms underpinning computer predictions in modern systems science. The course will cover the classical fundamental topics in numerical methods such as, numerical integration, solution of ordinary and partial differential equations. Linear operators, multi-step methods and solution of difference equations of different types. The viewpoint will be modern, with connections made between each topic and a variety of applications.

Contents

- 1 Numerical differentiation and Numerical Integration: Numerical Differentiation: Introduction, Differentiation using difference operators, Differentiation using interpolation
- 2 Numerical Integration: Newton-Cotes integration formulae, Composite Integration, Rectangular rule, Trapezoidal rule (simple and composite), Simpson's 1/3 rule and Simpson's 3/8 rule of integration
- 3 Boole's rule & Weddle's rules, Basic technique of finding error term
- 4 Numerical Solution of ODE: Introduction, Initial Value Problem, Boundary Value Problem, Taylor's Series method, Euler method, modified Euler's method, Instability criteria of Euler method, Runge-Kutta methods for solving initial value problems
- 5 Multi step methods: Predictor-Corrector method, Milne's methods, Adam Bash-forth methods
- 6 Difference Equations: Introduction to numerical solutions, Formulation of difference equations, Solution of linear/non-linear difference equations with constant coefficients, Solution of homogeneous difference equations with constant coefficients, Solution of inhomogeneous difference equations with constant coefficients

Pre-requisite: Numerical Analysis-I

Recommended Texts

1. Burden, R. L., Faires, J. D., & Burden, A.M. (2015). *Numerical analysis* (10th ed.). Boston: Cengage Learning.
2. Gerald, C. F., & Wheatley, P.O. (2003). *Applied numerical analysis* (7th ed.). London: Pearson.

Suggested Readings

1. Kuo, Shan S. (1972). *Computer applications of numerical methods*. Islamabad: National Book Foundations.
2. Philip, J. (2019). *Numerical applied computational programming with case studies* (1st ed.). New York: A press.
3. Khoury, R., & Harder, D.W. (2016). *Numerical methods & modelling for engineering* (1st ed.). London: Springer.
4. Antia, H.M. (2012). *Numerical methods for scientists & engineers* (3rd ed.). New York: Springer.

Department of Mathematics
University of Chittagong

Mathematical methods present an applied mathematics course designed to provide the necessary analytical and numerical background for courses in astrophysics, plasma physics, fluid dynamics, electromagnetism, and radiation transfer. The main objective of this course is to provide the students with a range of mathematical methods that are essential to the solution of advanced problems encountered in the fields of applied physics & engineering. Calculation-oriented mathematics is included in all topics relevant. Systems of linear equations, Gauss-Jordan-elimination, basic matrix algebra, determinants. Limits and continuity, differentiation and integration of functions in one variable, maxima and minima, implicit differentiation and trigonometric functions, related rates, differentials and linearization, L'Hopital's rule, Newton's method and the bisection method. Riemann sums and the fundamental theorem in calculus, integral functions, definite and indefinite integrals, basic integration techniques, substitution and partial integration, numerical integration by the rectangle and trapezium methods, improper integrals. Area, volume and arc length. Modeling with differential equations, first order separable and linear differential equations, Euler's method, second order linear differential equations with constant coefficients.

Contents

- 7 Hankel transforms for the solution of PDEs & their application to boundary value problems
- 8 Green's Functions & Transform Methods: Expansion for Green's functions
- 9 Sturm-Liouville problems: Introduction to eigen value problem, adjoint & self-adjoint operators,
- 10 Self-adjoint differential equations, eigen values & eigen functions
- 11 Sturm-Liouville (S-L) boundary value problems, regular & singular S-L problems
- 12 Transform methods. Closed form Green's functions. Perturbation Techniques
- 13 Perturbation methods for algebraic equations
- 14 Perturbation methods for differential equations
- 15 Variational Methods: Euler-Lagrange equations
- 16 Integr & involving one, two, three & n variables
- 17 Special cases of Euler-Lagrange's equations
- 18 Necessary conditions for existence of an extremum of a functional
- 19 Constrained maxima & minima

Recommended Texts

1. Powers, D. L. (2005). *Boundary value problems & partial differential equations* (5th ed.). Boston: Academic Press.
2. Boyce, W.E. (2005). *Elementary differential equations* (8th ed.). New York: John Wiley & Sons.

Suggested Readings

1. Brown, J.W., & Churchill, R.V. (2006). *Fourier series & boundary value problems*. New York: McGraw Hill.
2. Snider, A.D. (2006). *Partial differential equations*. New York: Dover Publications Inc.
3. Boyce, W.E. (2005). *Elementary differential equations* (8th ed.). New York: John Wiley & Sons.
4. Krasnov, M.L., Makarenko, G.I., & Kiselev, A.I. (1985). *Problems & exercises in the calculus of variations*. USA: Imported Publications, Inc.



Differential geometry is the study of geometric properties of curves, surfaces, & their higher dimensional analogues using the methods of calculus. It has a long & rich history, &, in addition to its intrinsic mathematical value & important connections with various other branches of mathematics, it has many applications in various physical sciences, e.g., solid mechanics, computer tomography, or general relativity. Differential geometry is a vast subject. This course covers many of the basic concepts of differential geometry in the simpler context of curves & surfaces in ordinary 3-dimensional Euclidean space. The aim is to build both a solid mathematical understanding of the fundamental notions of differential geometry & enough visual & geometric intuition of the subject. This course is of interest to students from a variety of math, science & engineering backgrounds, & that after completing this course, the students will be ready to study more advanced topics such as global properties of curves & surfaces, geometry of abstract manifolds, tensor analysis, & general relativity.

Contents

- 1 Space Curves
- 2 Arc length, tangent
- 3 Normal & binormal
- 4 Curvature & torsion of a curve
- 5 Tangent planes
- 6 The Frenet-Serret apparatus
- 7 Fundamental existence theorem of plane curves
- 8 Four vertex theorem, Isoperimetric inequality
- 9 Surfaces
- 10 First fundamental form
- 11 Isometry & conformal mappings
- 12 Curves on Surfaces, surface Area
- 13 Second fundamental form
- 14 Normal & Principle curvatures
- 15 Gaussian & Mean curvatures
- 16 Geodesics

Recommended Texts

1. Somasundaran, D. (2005). *Differential geometry* (1st ed.). New Delhi: Narosa Publishing House.
2. Pressley, A. (2001). *Elementary differential geometry* (1st ed.). New York: Springer-Verlag.

Suggested Readings

1. Wilmore, T. J. (1959). *An introduction to differential geometry* (1st ed.). Oxford: Clarendon Press.
2. Weatherburn, C. E. (2016). *Differential geometry of three dimensions*. Cambridge University Press.
3. Millman, R. S., & Parker, G. D. (1977). *Elements of differential geometry*. Englewood Cliffs: Prentice Hall.



Many physical problems that are usually solved by differential equation methods can be solved more effectively by integral equation methods. This course will help students gain insight into the application of advanced mathematics & guide them through derivation of appropriate integral equations governing the behavior of several standard physical problems. In addition, a large class of initial & boundary value problems, associated with the differential equations, can be reduced to the integral equations, whence enjoy the advantage of the above integral presentations. This course has many applications in many sciences. This course emphasizes concepts and techniques for solving integral equations from an applied mathematics perspective. Material is selected from the following topics: Volterra and Fredholm equations, Fredholm theory, the Hilbert-Schmidt theorem; Wiener-Hopf Method; Wiener-Hopf Method and partial differential equations; the Hilbert Problem and singular integral equations of Cauchy type; inverse scattering transform; and group theory. Examples are taken from fluid and solid mechanics, acoustics, quantum mechanics, and other applications.

Contents

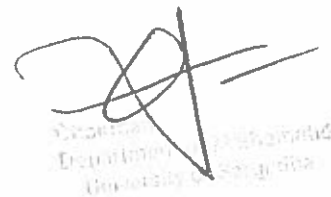
- 1 Linear integral equations of the first kind
- 2 Linear integral equations of the second kind
- 3 Relationship between differential equation & Volterra integral equation
- 4 Neumann series
- 5 Fredholm Integral equation of the second kind with separable Kernels
- 6 Eigen values, Eigenvectors
- 7 Iterated functions
- 8 Quadrature methods
- 9 Least square methods
- 10 Homogeneous integral equations of the second kind
- 11 Fredholm integral equations of the first kind
- 12 Fredholm integral equations of the second kind
- 13 Abel's integral equations
- 14 Hilbert Schmidt theory of integral equations with symmetric Kernels
- 15 Regularization & filtering techniques

Recommended Texts

- 1 Jerri, J. (2007). *Introduction to integral equations with applications* (2nd ed.). New York: Sampling Publishing,
- 2 Wazwaz, A.M. (2011). *Linear & nonlinear integral equations: methods & applications*. New York: Springer.

Suggested Readings

- 1 Lovitt, W.V. (2005). *Linear integral equations*. New York: Dover Publications.
- 2 Christian, C., Dale, D., & Hamill, W. (2014). *Boundary integral equation methods & numerical solutions* (1st ed.). New York: Springer.
- 3 Kanwal, R. P. (1996). *Linear integral equations: theory & technique*. Boston: Birkhauser
- 4 Tricomi, F. G. (1985). *Integral Equations*. New York: Dover Pub.



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This course extends methods of linear algebra & analysis to spaces of functions, in which the interaction between algebra & analysis allows powerful methods to be developed. The course will be mathematically sophisticated & will use ideas both from linear algebra & analysis. This is a basic graduate level course that introduces the student to Functional Analysis & its applications. It starts with a review of the theory of metric spaces, the theory of Banach spaces & proceeds to develop some key theorems of functional analysis. Then continuous to linear operators in Banach & Hilbert spaces & to spectral theory of self-adjoint operators with applications to the theory of boundary value problems, & the theory of linear elliptic partial differential equations.

Contents


- 1 Metric Spaces
- 2 Completeness of metric space
- 3 Completeness proofs
- 4 Normed linear Spaces, Banach Spaces
- 5 Equivalent norms
- 6 Finite dimensional normed spaces, Linear operator
- 7 Continuous & bounded linear operators
- 8 Linear functional, Dual spaces
- 9 Functional on finite dimensional Spaces
- 10 Inner product Spaces
- 11 Hilbert Spaces
- 12 Conjugate spaces
- 13 Representation of linear functional on Hilbert space
- 14 Orthogonal sets
- 15 Orthonormal sets & sequences
- 16 Orthogonal complements & direct sum
- 17 Reflexive spaces

Recommended Texts

- 1 Kreyszig, E. (1989). *Introduction to functional analysis with applications* (1st ed.). New York: John Wiley & Sons.

Suggested Readings

- 1 Dunford, N., & Schwartz, J. T. (1958). *Linear operators, part-1 general theory*. New York: Interscience publishers.
- 2 Balakrishnan, A. V. (1981). *Applied functional analysis* (2nd ed.). New York: Springer-Verlag.
- 3 Conway, J. B. (1995). *A Course in functional analysis* (2nd ed.). New York: Springer-Verlag.



Special functions are particular mathematical functions that have more or less established names & notations due to their importance in mathematical analysis, functional analysis, geometry, physics, or other applications. The term is defined by consensus, & thus lacks a general formal definition, but the List of mathematical functions contains functions that are commonly accepted as special. The main aim of this course is the study of basic special functions & proves the properties & relations related to these functions. Furthermore, the simple sets of polynomials are discussed.

Contents

- 1 The Weierstrass gamma function
- 2 Euler integral representation of gamma function
- 3 Relations satisfied by gamma function
- 4 Euler's constant
- 5 Properties of gamma function
- 6 Beta function, integral representation of beta function
- 7 Relation between gamma & beta functions
- 8 Properties of beta function, Legendre's duplication formula
- 9 Gauss' multiplication theorem
- 10 Hypergeometric series, the functions $F(a,b;c;z)$ & $F(a,b;c;l)$, integral representation of hypergeometric function,
- 11 The hypergeometric differential equation, The contiguous relations, Simple transformations,
- 12 A theorem due to Kummer,
- 13 Differential equation, Kummer's first formula
- 14 Simple sets of polynomials, Orthogonality,
- 15 The three term recurrence relation, The Christoffel-Darboux formula,
- 16 Bessel Functions
- 17 Generating functions

Recommended Texts

1. Richard, B. (2016). *Special functions & orthogonal polynomials*. Cambridge: Cambridge University Press.
2. Rainville, E. D. (1971). *Special functions* (3rd ed.). New York: The Macmillan Company

Suggested Readings

1. Whittaker, E. T., & Watson, G. N. (1978). *A course in modern analysis*, (2nd ed.). Cambridge: Cambridge University Press.
2. Lebedev, N. N. (1972). *Special functions & their applications* (2nd ed.). New York: Prentice Hall.



The objective of this course is to understand & apply the fundamental concepts in graph theory, apply graph theory-based tools in solving practical problems & to improve the proof writing skills. Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among others fields, to solve problems that can be modeled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including the group theory, the matrix theory, the numerical analysis, probability, topology, & the combinatorics. Even though some of the problems in graph theory can be described in an elementary way, many of these problems represent a challenge to many researchers in mathematics. The main focus of this course is to understand & apply the fundamental concepts in graph theory. To apply graph theory-based tools in solving practical problems. To improve the proof writing skills.

Contents

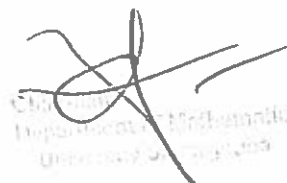
- 1 Graphs & digraphs
- 2 Degree sequences
- 3 Paths
- 4 Cycles, cut-vertices, & blocks
- 5 Eulerian graph
- 6 Digraphs
- 7 Trees
- 8 Incidence matrix
- 9 Cut-matrix
- 10 Circuit matrix & adjacency matrix
- 11 Orthogonality relation
- 12 Decomposition
- 13 Euler formula
- 14 Planer graphs
- 15 Non-planer graphs
- 16 Mengers theorem
- 17 Hamiltonian's graphs

Recommended Texts

1. Chartrand, G., Lesniak, L., & Zhang, P. (2010). *Graphs & digraphs* (5th ed.). Florida: Chapman & Hall.
2. Ruohonen, K. (2013). *Graph theory* (translation by Janne Tamminen, Kung-Chung Lee & Robert Piché). http://math.tut.fi/~ruohonen/GT_English.pdf

Suggested Readings

1. Robin, J. W. (1996). *Introduction to graph theory* (4th ed.). Boston: Addison Wesley.
2. Bondy, J. A., & Murty, S. U. R. (1976). *Graph theory with applications*. United States: The Macmillian Press Ltd.



Chapman & Hall
Mathematics Department
University of Cambridge

This is the first part of the two advance course series of Group Theory. This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. In general, however, there is no hope of a similar result as the situation is far too complex, even for finite groups. Still, since groups are of great importance for the whole of mathematics, there is a highly developed theory of outstanding beauty. It takes just three simple axioms to define a group, & it is fascinating how much can be deduced from so little. The course is devoted to some of the basic concepts & results of Group Theory.


Contents

- 1 Group of automorphisms, inner automorphisms, definition & related results
- 2 Characteristic & fully invariant subgroups,
- 3 Symmetric Groups, cyclic permutations
- 4 Even & odd permutations
- 5 The alternating groups, conjugacy classes of symmetric & alternating groups
- 6 Generators of symmetric & alternating groups
- 7 Simple groups
- 8 Simplicity of symmetric & alternating groups
- 9 Group Action on sets or G-sets
- 10 Orbits & stabilizer subgroups
- 11 Finite direct products
- 12 Finitely generated abelian groups
- 13 P-groups, Sylow's Theorems
- 14 Application of Sylow's Theorems
- 15 Linear Groups

Recommended Texts

1. Rotman, J. J. (1999). *An Introduction to the theory of groups* (4th ed). New York: Springer.
2. Shah, S. K., & Shankar A. G. (2013). *Group theory*. London: Dorling Kindersley.
3. Dummit, D.S. Foote, R.M. (2003). *Abstract Algebra* (3rd Ed.), USA: Wiley.

Suggested Readings

1. Rose, H. E. (2009). *A course on finite groups* (1st ed). New York: Springer-Verlag.
 2. Fraleigh, J. B. (2003). *A first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.
 3. Malik, D. S., Mordeson J. N., & Sen M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.
 4. Rose, J. A. (2012). *Course on group theory* (Revised ed.). New York: Dover Publications.
- 

This course is the continuation of the course "Advanced Group Theory-1". This course aims to introduce students to some more sophisticated concepts & results of group theory as an essential part of general mathematical culture & as a basis for further study of more advanced mathematics. The ideal aim of Group Theory is the classification of all groups (up to isomorphism). It will be shown that this goal can be achieved for finitely generated abelian groups. This course covers the advanced topics in group theory such as solvable groups, Upper & Lower Central series nilpotent groups & free groups.

Contents

- 1 Series in groups
- 2 Normal series
- 3 Normal series & its refinement
- 4 Composition series
- 5 Equivalent composition series
- 6 Jordan Holder Theorem
- 7 Solvable groups, definition, examples & related results
- 8 Upper & Lower Central series
- 9 Nilpotent groups
- 10 Characterization of finite nilpotent groups
- 11 The Frattini subgroups, definition, examples & related results
- 12 Free groups, definition, examples & related results
- 13 Free Product, definition, examples & related results
- 14 Group algebras

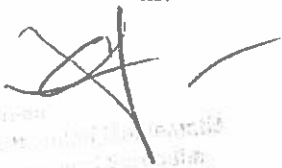
Pre-requisite: Advance Group Theory-I

Recommended Texts

1. Rotman, J. J. (1999). *An Introduction to the theory of groups* (4th ed). New York: Springer.
2. Shah, S.K., & Shankar A. G. (2013). *Group theory*. London: Dorling Kindersley.
3. Dummit, D.S. Foote, R.M. (2003). *Abstract Algebra* (3rd Ed.), USA: Wiley.

Suggested Readings

1. Rose, H. E. (2009). *A course on finite groups* (1st ed). New York: Springer-Verlag.
2. Fraleigh, J. B. (2003). *A first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.
3. Malik, D. S., Mordeson J. N., & Sen M. K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.
4. Rose, J. A. (2012). *Course on group theory* (Revised ed.). New York: Dover Publications.


Vishwanath
Department of Mathematics
American University of Sharjah

This course is an introduction to module theory, for those who know something about linear algebra and ring theory. Its main aim is the derivation of the structure theory of modules over Euclidean domains. This theory is applied to obtain the structure of abelian groups and the rational canonical and Jordan normal forms of matrices. The basic facts about rings and modules are given in full generality, so that some further topics can be discussed, including projective modules and the connection between modules and representations of groups. It aims to develop the general theory of rings & then study in some detail a new concept, that of a module over a ring. The theory of rings & module is key to many more advanced algebra courses. This subject presents the foundational material for the rest of the basic algebraic structure pervading contemporary pure mathematics, namely fields & modules. The basic definitions & elementary results are given, followed by two important applications of the theory. This course introduces concepts of modules. The main objective of this course is to prepare students for courses which require a good background in Modules Theory, Primary component & Invariance Theorem etc.

Contents

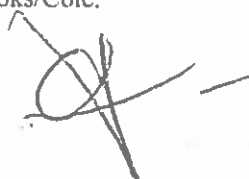
- 1 Polynomial rings
- 2 Division algorithm for polynomials
- 3 Prime elements, Irreducible elements
- 4 Euclidean domain
- 5 Principal ideal domain
- 6 Greatest common divisor
- 7 Unique factorization domain
- 8 Factorization of polynomials over a UFD
- 9 Irreducibility of polynomials
- 10 Eisenstein's irreducibility criterion
- 11 Maximal ideals, Prime ideals, Primary ideals
- 12 Noetherian rings, Artinian rings
- 13 Modules, sub modules, Quotient modules
- 14 Finitely generated & cyclic modules, Exact sequences
- 15 Elementary notions of homological algebra, Noetherian modules
- 16 Artinian modules, Radicals, Semisimple modules
- 17 Tensor product of modules, Bimodules
- 18 Algebra & coalgebra, Torsion module
- 19 Primary components, Invariance theorem

Recommended Texts

1. Wang, F., & Kim, H. (2016). *Foundations of commutative rings & their modules* (1st ed.). New York: Springer.
2. Berrick, A. J., & Keating, M. E. (2000). *An introduction to rings & modules: With K-Theory in View* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1. Hartley, B., & Hawkes, T. O. (1980). *Rings, modules & linear algebra* (1st ed.). London: Chapman & Hall.
2. Herstein I. N. (1995). *Topics in algebra with application* (3rd ed.). New York: Books/Cole.
Blyth, T. S. (1977). *Module theory* (1st ed.). Oxford: Oxford University Press.



This course will cover basics of abstract rings and fields, which are an important part of any abstract algebra course sequence. We will begin with definitions and important examples. We discuss extension of fields, adjoining roots, and prove the primitive element theorem. Finally, we will classify finite fields. Rings are one of the fundamental languages of mathematics & they play a key role in many areas, including algebraic geometry, number theory, Galois theory & representation theory.

Contents

- 1 Extension fields
- 2 Finite fields, fields extension,
- 3 Algebraic elements
- 4 Transcendental elements
- 5 Simple extension
- 6 Galois theory,
- 7 Galois theory of equations,
- 8 Construction with straight-edge and compass,
- 9 Splitting field of polynomials,
- 10 The Galois groups,
- 11 Some results on finite groups,
- 12 Symmetric group as Galois group,
- 13 Constructable regular n-gones,
- 14 The Galois group as permutation group.

Recommended Texts

1. Cohn, P. M. (2006). *Free ideal rings & localization in general*. Cambridge: Cambridge University Press.
2. Lang, S. (2005). *Algebra*. Boston: Addison Wesley.

Suggested Readings

1. Herstein, I. N. (1975). *Topics in algebra*. New York: John Wiley & Sons Inc.
2. Hartley, B., & Hawkes, T. O. (1970). *Ring, modules & linear algebra*. Florida: Chapman & Hall
3. Fraleigh, J. A. (1982). *A first course in abstract algebra*. Boston: Addison Wesley.
4. Roman, S. (2005). *Field theory: Graduate texts in mathematics (2nd ed)*. Berlin: Springer.


Department of Mathematics
University of Colorado

This course is the first part of the core level course on fluid mechanics. Fluid mechanics is the branch of physics concerned with the mechanics of fluids (liquids, gases, & plasmas) & the forces on them. It has applications in a wide range of disciplines, including mechanical, civil, chemical & biomedical engineering, geophysics, oceanography, meteorology, astrophysics, & biology. The course of fluid mechanics is introducing fundamental aspects of fluid flow behavior. Students will learn properties of Newtonian fluids; apply concepts of mass, momentum & energy conservation to flows.

Contents

- 1 Introduction: Definition of Fluid, basics equations
- 2 Methods of analysis, dimensions & units, Fundamental concepts
- 3 Fluid as a continuum, velocity field, stress field, viscosity, surface tension, description & classification of fluid motions
- 4 Fluid Statics: The basic equation of fluid static
- 5 The standard atmosphere
- 6 Pressure variation in a static fluid
- 7 Fluid in rigid body motion. Basic equation in integral form for a control volume
- 8 Basic laws for a system
- 9 Relation of derivatives to the control volume formulation
- 10 Conservation of mass
- 11 Momentum equation for inertial control volume
- 12 Momentum equation for control volume with rectilinear acceleration
- 13 Momentum equation for control volume with arbitrary acceleration
- 14 The angular momentum principle
- 15 The first law of thermodynamics
- 16 The second law of thermodynamics
- 17 Introduction to differential analysis of fluid motion
- 18 Conservation of mass
- 19 Stream function for two-dimensional incompressible flow
- 20 Motion of a fluid element (kinematics), momentum equation

Recommended Texts

1. Fox, R. W., & McDonald, A. T. (2004). *Introduction to fluid mechanics* (6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: Mc. Graw Hill.

Suggested Readings

1. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). Montana: Winston Publisher.
2. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey Son Publisher.
3. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Oxford: Butterworh Heinemann Publisher.



This course is the second part of the core level course on fluid mechanics. Fluid mechanics is concerned with the mechanics of fluids (liquids, gases, & plasmas) & the forces on them. This course covers properties of fluids, laws of fluid mechanics & energy relationships for incompressible fluids. Studies flow in closed conduits, including pressure loss, flow measurement, pipe sizing & pump Selection, momentum equation for frictionless flow, Euler's equations, Bernoulli equation- Integration of Euler's equation, laminar flow & Boundary layers.

Contents

- 1 Incompressible inviscid flow
- 2 Momentum equation for frictionless flow
- 3 Euler's equations
- 4 Euler's equations in streamline coordinates
- 5 Bernoulli equation- Integration of Euler's equation along a streamline for steady flow
- 6 Relation between first law of thermodynamics & the Bernoulli equation
- 7 Unsteady Bernoulli equation-Integration of Euler's equation along a streamline
- 8 Irrotational flow, internal incompressible viscous flow
- 9 Fully developed laminar flow
- 10 Fully developed laminar flow between infinite parallel plates
- 11 Fully developed laminar flow in a pipe
- 12 Part-B Flow in pipes & ducts
- 13 Shear stress distribution in fully developed pipe flow
- 14 Turbulent velocity profiles in fully developed pipe flow
- 15 Energy consideration in pipe flow
- 16 External incompressible viscous flow
- 17 Boundary layers, the boundary concept, boundary thickness, laminar flat plate
- 18 Boundary layer: exact solution, momentum, integral equation,
- 19 Use of momentum integral equation for flow with zero pressure gradient
- 20 Pressure gradient in boundary-layer flow

Pre-requisite: Fluid Mechanics-I

Recommended Texts

1. Fox, R. W., & McDonald, A. T. (2004). *Introduction to fluid mechanics* (6th ed.). New York: John Wiley & Sons.
2. White, F. M. (2006). *Fluid mechanics* (5th ed.). New York: Mc. Graw Hill.

Suggested Readings

1. Bruce, R., Rothmayer, A. P., Theodore, H. O., & Wade, W. H. (2013). *Fundamental of fluid mechanics* (7th ed.). New York: Willey Son Publisher.
2. Nakayama, Y. (2018). *Introduction to fluid mechanics* (2nd ed.). Oxford: Butterworh Heinemann Publisher.
3. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). Montana: Winston Publisher.



This course is the 1st part of the course series on operation research. Operations research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. Operations Research studies analysis and planning of complex systems. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real-life decision-making problems. The purpose of the course is to provide students with the concepts and tools to help them understand the operations research and mathematical modeling methods and to understand different application areas of operations research like transportation problem, assignment model, sequencing models, dynamic programming, game theory, replacement models & inventory models.

Contents

- 1 Linear Programming
- 2 Formulation & graphical solution
- 3 Simplex method, M-technique
- 4 Two-phase technique
- 5 Special cases
- 6 Sensitivity analysis
- 7 The dual problem
- 8 Primal dual relationship
- 9 The dual simplex method
- 10 Sensitivity
- 11 Post optimal analysis
- 12 Transportation model
- 13 Northwest corner
- 14 Least cost
- 15 Vogel's approximation methods
- 16 The method of multipliers
- 17 The assignment models
- 18 The transshipment model
- 19 Network minimization
- 20 Shortest route algorithms for variables

Recommended Texts

1. Hamdy, A. T. (2006). *Operations research an introduction* (6th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1st ed.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research: A practical introduction* (1st ed.). North Holland: CRC Press
2. Ravindran, A. R. (2008). *Operations research applications* (1st ed.). North Holland: CRC Press.



Operations Research (OR) is an analytical method of problem-solving & decision-making that is useful in the management of organizations. In operations research, problems are broken down into basic components & then solved in defined steps by mathematical analysis. Disciplines that are similar to, or overlap with, operations research include statistical analysis, management science, game theory, optimization theory, artificial intelligence & network analysis. All of these techniques have the goal of solving complex problems & improving quantitative decisions. The objective of Operations Research, as a mathematical discipline, is to establish theories & algorithms to model & solve mathematical optimization problems that translate to real life decision making problems. Students would be able to identify & develop complicated operational research modals from the verbal description of the real system. The understanding of the mathematical tools that are needed to solve optimization problems would be increased. They would be analyze the results & propose the theoretical language understandable to decision making processes in Management Engineering.

Contents

- 1 Algorithm for cyclic network
- 2 Maximal flow problems
- 3 Matrix definition of LP- problems
- 4 Revised simplex methods
- 5 Bounded variables decompositions algorithm
- 6 Parametric linear programming
- 7 Application of integer programming
- 8 Cutting plane algorithm
- 9 Mixed fractional cut algorithm
- 10 Branch methods
- 11 Bound methods
- 12 Zero-one implicit enumeration
- 13 Element of dynamics programming
- 14 Problems of dimensionality
- 15 Solutions of linear program by dynamics programming

Pre-requisite: Operation Research-I

Recommended Texts

1. Hamdy, A. T. (2006). *Operations research an introduction* (6th ed.). New York: Macmillan.
2. Gillet, B. E. (1979). *Introduction to operations research* (1st ed.). New York: McGraw Hill.

Suggested Readings

1. Harvy, C. M. (1979). *Operations research: A practical introduction* (1st ed.). North Holland: CRC Press



In classical mechanics, analytical dynamics, or more briefly dynamics, is concerned with the relationship between motion of bodies & its causes, namely the forces acting on the bodies & the properties of the bodies, particularly mass & moment of inertia. Analytical dynamics develops Newtonian mechanics to the stage where powerful mathematical techniques can be used to determine the behavior of many physical systems. The mathematical framework also plays a role in the formulation of modern quantum & relativity theories.

Contents

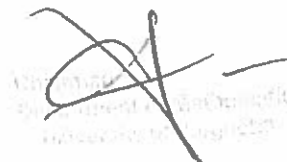
- 1 Generalized coordinates
- 2 Constraints
- 3 Degree of freedom
- 4 D'Alembert principle
- 5 Holonomic & non-Holonomic systems, Hamilton's principle
- 6 Derivation of Lagrange equation from Hamilton's principle
- 7 Derivation of Hamilton's equation from a variational principle
- 8 Equations & Examples of Gauge transformations
- 9 Equations & examples of canonical transformations
- 10 Orthogonal Point transformations
- 11 The Principle of Least Action
- 12 Applications of Hamilton's equation to central force problems
- 13 Applications to Harmonic oscillator
- 14 Hamiltonian formulism
- 15 Lagrange bracket & Poisson brackets with application
- 16 The Hamilton Jacobi theory, Hamilton Jacobi Theorem
- 17 The Hamilton Jacobi equation for Hamilton characteristic functions
- 18 Bilinear co-variant

Recommended Texts

1. Greenwood, D. T. (1965). *Classical dynamics*. New Jersey: Prentice-Hall, Inc.
2. Aruldas, G. (2016). *Classical mechanics*. New Dehli: PHI Private Limited.
3. Chorlton, F. (1983). *Textbook of dynamics*. Cambridge: E. Horwood.

Suggested Readings

1. Woodhouse, N. M. J. (2009). *Introduction to analytical dynamics* (2nd ed.). New York: Springer-Verlag.
2. Chester, W. (1979). *Mechanics*. London: New South Wales: George Allen & Unwin Ltd.



This course introduces the basic ideas and equations of Einstein's Special Theory of Relativity to understand the physics of Lorentz contraction, time dilation, the "twin paradox", and $E=mc^2$. Calculus Vector transformations Tensors for GTR to understand why we need these two theories. For that see the problems with Galilean transformation & equivalence of inertial & gravitational mass. The most important thing to study SR is to accept geometry as the concept behind it. The math is not difficult; it's the way of thinking you have to adopt. Draw space time diagrams, something to transform to another frame of reference (Lorentz transforms are available). Keep in mind that the view in the other reference frame is just a different view of the same situation that nothing really has changed, even if it looks different on Euclidean paper.

Contents

- 1 Historical background
- 2 Fundamental concepts of special theory of relativity
- 3 Galilean transformations,
- 4 Lorentz transformations (for motion along one axis)
- 5 Length contraction
- 6 Time dilation
- 7 Simultaneity
- 8 Velocity addition formulae.3-dimensional
- 9 Lorentz transformations
- 10 Introduction to 4-vector formalism
- 11 Lorentz transformations in the 4-vector formalism
- 12 Minkowski space-time & null cone
- 13 4-velocity & 4-momentum & 4-force
- 14 Application of special relativity to Doppler shift & Compton effect
- 15 Aberration of light
- 16 Particle scattering, Binding energy
- 17 Particle production & decay

Recommended Texts

1. Qadir, A. (1989). *An introduction to the special relativity theory* (1st ed.). Singapore: World Scientific.
2. Sardesai, P.L. (2008). *A primer of special relativity* (2nd ed.). Delhi: Offset.

Suggested Readings

1. Resnick, R. (1968). *Introduction to special relativity*. New York: Wiley.
2. D'Inverno, R. (1992). *Introducing Einstein's relativity* (1st ed.). Oxford: Oxford University Press.



Heat transfer is a discipline of thermal engineering that concerns the generation, use, conversion, & exchange of thermal energy (heat) between physical systems. Heat transfer is classified into various mechanisms, such as thermal conduction, thermal convection, thermal radiation, & transfer of energy by phase changes. The objectives of heat transfer include the following: Insulation, wherein across a finite temperature difference between the system & its surrounding, the engineer seeks to reduce the heat transfer as much as possible. The learning outcomes of this course are: to explain the basics of heat transfer, to explain the importance of heat transfer, to define the concept of boiling & condensation, to define the concept of heat exchangers, to explain heat transfer by conduction, to explain the Fourier heat conduction law, to define thermal conductivity coefficient & diffusion coefficient, to explain heat transfer with convection, to explain Newton's law, to explain free transport phenomenon, to explain the forced convection, to explain heat transfer by radiation.

Contents


- 1 Steady-State Conduction-One Dimension
- 2 Steady-State Conduction-Multiples Dimensions
- 3 Unsteady-State Conduction,
- 4 Principles of Convection
- 5 Empirical & practical Relations
- 6 Forced -Convection Heat Transfer
- 7 Natural Convection Systems
- 8 Radiation Heat Transfer

Recommended Texts

1. Holman, J. P. (1996). *Heat transfer* (8th ed.). New York: McGraw Hill.
2. Kays, W. M., & Crawford, M. E. (1993). *Convective heat & mass transfer* (3rd ed.). New York: McGraw Hill.

Suggested Readings

1. Incropera, F. P., & Dewitt, D. P. (1985). *Fundamentals of heat & mass transfer* (2nded). New York: Wiley.
2. Cengel, Y., & Ghajar, A. J. (2015). *Heat & mass transfer: Fundamentals & applications* (5th ed.). New York: Mc-Graw Hill.
3. Lienhar IV, J. H., & Lienhar V, J. H. (2019). *A heat transfer textbook* (5th ed.). New York: Dover Publications.
4. Incropera, F. P. (2006). *Fundamentals of heat & mass transfer* (6th ed.). New York: John Wiley & Sons.

A handwritten signature in black ink is written over a faint, circular stamp. The stamp contains some illegible text, possibly a name or title, but it is too light to read clearly.

The objectives of the course are to introduce the concepts of measure & integral with respect to a measure, to show their basic properties, to provide a basis for further studies in analysis, probability, & dynamical Systems, to construct Lebesgue's measure & learn the theory of Lebesgue integrals on real line & in n -dimensional Euclidean space. The goal of the course is to develop the understanding of basic concepts of measure and integration theory. As measure theory is a part of the basic curriculum since it is crucial for understanding the theoretical basis of probability and statistics, so it is intended to develop understanding of the theory based on examples of application. After the course the students will know & understand the basic concepts of measure theory & the theory of Lebesgue integration. The students will understand the main proof techniques in the field & will also be able to apply the theory abstractly & concretely. The students will be able to write the elementary proofs themselves, as well as more advanced proofs under guidance. The students will be able to use measure theory & integration in Riemann integration & calculus.

Contents

- 1 Introduction to Lebesgue measure
- 2 Outer measure
- 3 Properties of outer measure
- 4 Further properties of outer measure
- 5 Measurable sets
- 6 Properties of measurable sets
- 7 Non measurable sets
- 8 Measurable functions
- 9 Properties of measurable functions
- 10 Convergence of sequences of measurable functions
- 11 Lebesgue integration, introduction
- 12 Lebesgue integrals of simple functions
- 13 Bounded functions
- 14 Lebesgue integrals of non-negative functions
- 15 Lebesgue integration of general functions
- 16 General convergence theorems
- 17 convergence in measure

Recommended Texts

1. Roydon, H. L., & Fitzpatrick, P. M. (2017). *Real analysis* (4thed.). New York: Collier Macmillan Co.
2. Barra, G. D. (1981). *Measure theory & integration* (1st ed.). Ellis: Harwood Ltd.

Suggested Readings

1. Rudin, W. (1987). *Real & complex analysis*, (3rded.). New York: McGraw Hill Book Company.
2. Bartle, R.G. (1995). *The elements of integration & Lebesgue measure* (1sted.). Wiley-Interscience.
3. Halmos, P. R. (1975). *Measure theory* (1st ed.). New York: Springer.



This is the first part of the two-course series of Theory of Splines. This course is designed to teach students about basics of scientific computing for solving problems which are generated by data using interpolation & approximation techniques & learn how to match numerical method to mathematical properties. This course gives the students the knowledge of problem classes, basic mathematical & numerical concepts & software for solution of engineering & scientific problems formulated as using data sets. After successful completion, students should be able to design, implement & use interpolations for computer solution of scientific problems involving problems generated by set of data. The material covered provides the students with the necessary tools for understanding the many applications of splines in such diverse areas as approximation theory, computer-aided geometric design, curve and surface design and fitting, image processing, numerical solution of differential equations, and increasingly in business and the biosciences.

Contents

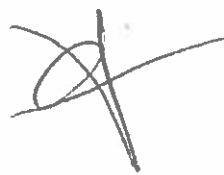
- 1 Basic concepts of Euclidean geometry
- 2 Scalar & vector functions
- 3 Barycentric coordinates
- 4 Convex hull, Matrices of affine maps, Translation, rotation, scaling
- 5 Reflection & shear, Curve fitting, least squares line fitting
- 6 Least squares power fit
- 7 Data linearization method for exponential functions
- 8 Nonlinear least-squares method for exponential functions
- 9 Transformations for data linearization
- 10 linear least squares, Polynomial fitting,
- 11 Basic concepts of interpolation, Lagrange's method,
- 12 Error terms & error bounds of Lagrange's method
- 13 Divided differences method,
- 14 Newton polynomials, error terms & error bounds of Newton polynomials
- 15 Central difference interpolation formulae
- 16 Gauss's forward interpolation formula
- 17 Gauss's backward interpolation formula, Hermite's methods

Recommended Texts

1. David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science + Business Media Inc.
2. John, H. M., & Kurtis, D. F. (1999). *Numerical methods using MATLAB*. New Jersey: Prentice Hall.

Suggested Readings

1. Rao, S. S. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.
2. Sudaran R. K. (1996). *A first course in optimization theory* (3rd ed.). Cambridge: Cambridge University Press.
3. Chang E. K. P., & Zak, S. I. I. (2004). *An introduction to optimization* (3rd ed.). New York: Wiley.



This is the second part of the two-course series of Theory of Splines. The goal of the course is to provide the students with a strong background on numerical approximation strategies & a basic knowledge on the theory of splines that supports numerical algorithms. Interactive graphics techniques for defining & manipulating geometrical shapes used in computer animation, car body design, aircraft design, & architectural design. In this course follow a modular approach & contribute different components to the development of an interactive curve & surface modeling system. Curve Modeling Techniques: Students will implement various curve interpolation & approximation techniques that allow the interactive specification of three-dimensional curves (e.g. Bezier, B-spline, rational curves). Surface modeling techniques: Students will implement various surface interpolation & approximation techniques that allow the interactive specification of three-dimensional surfaces (e.g. Bezier, B-spline, rational surfaces). Simple, 3D Modeling System: Students will integrate the curve & surface modules into a system that allows the user to interactively design & store simple, 3D geometries.

Contents

- 1 Parametric curves (scalar & vector case), Algebraic form
- 2 Hermite form, control point form, Bernstein Bezier form
- 3 Matrix forms of parametric curves
- 4 Algorithms to compute B.B. form, Convex hull property
- 5 Affine invariance property, Variation diminishing property
- 6 Rational quadratic form, Rational cubic form
- 7 Tensor product surface, B.B. cubic patch
- 8 Quadratic by cubic B.B. patch, B.B. quartic patch, Splines, Cubic splines
- 9 End conditions of cubic splines, Clamped conditions
- 10 Natural conditions, second derivative conditions
- 11 Periodic conditions, Not a knot conditions
- 12 General splines, Natural splines, Periodic splines
- 13 Truncated power function, Representation of spline in terms of truncated power functions
- 14 Odd degree interpolating splines

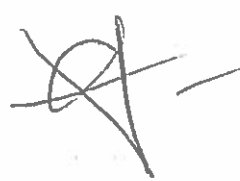
Pre-requisite: Theory of Splines-I

Recommended Texts

1. Farin, G. (2002). *Curves & surfaces for computer aided geometric design, a practical guide* (5th ed.). New York: Academic Press.
2. Faux, I. D., & Pratt, M. J. (1979). *Computational geometry for design & manufacture* (1st ed.). New York: Halsted Press.

Suggested Readings

1. Bartle, H. R., & Beatly, C. J. (2006). *An Introduction to spline for use in computer graphics & geometric modeling* (4th ed.). Massachusetts: Morgan Kaufmann.
2. Boor, C. D. (2001). *A practical guide to splines* (Revised ed.). New York: Springer Verlag.



Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. The objective of this course is to make students acquire a systematic understanding of optimization techniques. The course will start with linear optimization (being the simplest of all optimization techniques) and will discuss in detail the problem formulation and the solution approaches. Then we will cover a class of nonlinear optimization problems where the optimal solution is also globally optimal, i.e. convex nonlinear optimization and its variants. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

- 1 Introduction to optimization
- 2 Review of related mathematical concepts
- 3 Unconstrained optimization
- 4 Conditions for local minimizers
- 5 One dimensional search methods
- 6 Gradient methods
- 7 Newton's method (analysis & modifications)
- 8 Conjugate direction methods
- 9 Quasi Newton method
- 10 Application to neural network
- 11 Single Neuron Training
- 12 Linear integer programming
- 13 Genetic algorithms
- 14 Real number genetic algorithm

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory*, (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsitsiklis, J. (1997). *Introduction to linear optimization* (2nd ed.). Belmont: Athena Scientific



This is continuation of Methods of Optimization I. Optimization is a widely used technique in operational research that has been employed in a range of applications. The aim is to maximize or minimize a function (e.g. maximizing profit or minimizing environmental impact) subject to a set of constraints. At the start of the course, the course delivery, the prerequisites of the course will be discussed. Students will learn the foundations of linear programming, properties of optimal solutions and various solution methods for optimizing problems involving a linear objective function and linear constraints. Students will be exposed to geometric, algebraic and computational aspects of linear optimization and its extensions. On successful completion of the course the students will be able to model engineering maxima/minima problems as optimization problems. The students will be able to use computers to implement optimization algorithms. The students will learn efficient computation procedures to solve optimization problems.

Contents

- 1 Non-linear constrained optimization
- 2 Problems with equality constraints
- 3 Problem Formulation
- 4 Tangent spaces
- 5 Normal spaces
- 6 Lagrange condition
- 7 Second-order conditions
- 8 Problems with inequality constraints
- 9 Karush-Kuhn-Tucker Condition
- 10 Second-order conditions
- 11 Convex optimization problems
- 12 Convex functions
- 13 Algorithms for constrained optimization
- 14 Lagrangian algorithms

Pre-requisite: Methods of Optimization-I

Recommended Texts

1. Chong, E. K. P., & Stanislaw, H. Z. (2012). *An introduction to optimization* (4th ed.). New York: Wiley Series in Discrete Mathematics & Optimization.
2. Singiresu, S. R. (1992). *Optimization theory & applications* (2nd ed.). New York: Wiley Eastern Ltd.

Suggested Readings

1. Sundaram, R. K. (1996). *A first course in optimization theory*, (3rd ed.). Cambridge: Cambridge University Press.
2. Bertsimas, D., Tsitsiklis, J. N., & Tsitsiklis, J. (1997). *Introduction to linear optimization* (2nd ed.). Belmont: Athena Scientific.



The objective of this course is to meet the current and future needs for the interaction between mathematics and biological sciences. Mathematical modeling is being applied in every major discipline in the biomedical sciences. Some very different applications, and surprisingly successful, is in psychology, modeling of various human interactions, blood flow and functioning of different organs in human body.

Contents

- 1 An introduction to the use of continuous and discrete differential equations in the biological sciences,
- 2 Biological topics will include single species and interacting population dynamics,
- 3 modeling infectious and dynamic diseases,
- 4 Regulation of cell function,
- 5 Molecular interactions, neural and biological oscillators, and an introduction to biological pattern formation,
- 6 Mathematical tools such as phase portraits, bifurcation diagrams, perturbation theory,
- 7 Parameter estimation techniques that analyze and interpret biological models.

Recommended Books

- 1 Murray, J.D., *Mathematical Biology*, (Springer-Verlag, 2001).
- 2 Keener, J. and Sneyd, J., *Mathematical Physiology*, (Springer, New York, 1998).

Suggested Books

- 1 Murray, J.D., *Nonlinear Differential Equation Models in Biology*, (Clarendon Press, Oxford, 1977).



A handwritten signature in black ink is written over a faint, circular stamp. The stamp contains text that is mostly illegible but appears to include the words "Department of Mathematics" and "University of Toronto".

List of Interdisciplinary/Allied Courses

Programming Languages plays an important role in Mathematics. More often, the act of programming involves problem-solving in itself, where you then take your answers and apply them to build a program. However, mathematicians sometimes require some programming languages for assistance, and some of the best programming languages for math work wonders when you're trying to hone your skills and train yourself in a particular mathematical field. A number of computer software available to deal with mathematical computing & simulation. This course provides a practical introduction to most widely used Mathematical computing software's namely, MATHEMATICA or MAPLE. Maple has a fairly strong advantage when it comes to combinatorial math problems. It's also known for its functional programming constructs, making it extremely interesting to play around with. After this course students will be able to develop computer programs in this software according to their requirements in mathematical computing. It includes introduction to data-oriented Python packages, decision trees, support vector machines (SVM), neural networks, and machine learning.

Contents

Mathematica

- 1 Introduction to the basic environment of MATHMATICA & its syntax
- 2 Running MATHEMATICA
- 3 Numerical/Algebraic Calculations, vectors, Matrices, Sets, Lists, Tables, arrays
- 4 Symbolic Mathematics in MATHEMATICA
- 5 Functions & functional programming
- 6 Procedural programming, Do, for & while loops, Flow controls
- 7 Graphics, Plots of 2D & 3D functions, Packages within MATHEMATICA

Maple

1. Introductory Demonstration of Maple, symbolic computations in MAPLE
2. Vectors, Matrices, Sets, Lists, Tables, arrays & Arrays, Toolbars & Palettes
3. Operators, Constant, Elementary Functions, Procedures
4. If clauses, selection & conditional execution
5. Looping, for & while loop, looping commands, recursion
6. Plots of 2D & 3D functions, Packages within MAPLE

Recommended Texts

1. Wellin, P., Kamin, S., & Gaylord R. (2011). *An introduction to programming with mathematica*, (3rd ed.). Cambridge: Cambridge university press.
2. Monagan, M. B., & Geddes, K. O. (2005). *Maple introductory programming guide*. Waterloo: Maplesoft, a division of Waterloo Maple Inc.

Suggested Readings

1. Aladjev, V. Z., & Bogdivicus, M. A. (2006). *Maple: Programming, physical & engineering Problems*. London: Fultus Publishing.
2. Maeder, R. E. (1997). *Programming in mathematica* (3rd ed.). Boston: Addison-Weseley.
3. Hoste, J. (2009). *Mathematica demystified*. New York: McGraw Hill.



Mechanics is all about motion of a body. It deals with forces, motion and further to the laws of motion in inertial frames specifically. This course provides the students a broad understanding of the physical principles of the mechanics, to describe mechanical events that involve forces acting on macroscopic objects. The main objective of this course is to create quantitative skills in the students and to motivate them to think creatively and critically about scientific problems and experiments. Students are encouraged to share their thinking with teacher and the other students to examine different problem-solving strategies.

Contents

1. Motion in Two and Three Dimensions: Projectile Motion, Uniform Circular motion
2. Force and Motion: Newton's Laws, Some Particular Forces, Drag Force, Frictional Force, Terminal Speed
3. Kinetic Energy and work: Work done by Gravitational force, Work done by Spring Force, Work done by General variable force, Power
4. Potential Energy and Conservation of energy: Reading a potential energy curve, Work done on a system by an external force,
5. Center of Mass and Linear Momentum: center of mass, Newton's second law for a system of particles, linear momentum, two particle and many-particle systems, center of mass of solid objects, momentum changes in a system of variable mass. Collisions in the center-of-mass reference frame.

Experiments

1. To determine the value of 'g' by a compound pendulum
2. To determine the surface tension of water by capillary tube method
3. Determination of moment of inertia of a solid/hollow cylinder and a sphere etc.
4. To study the conservation of energy (Hook's Law)
5. To study the laws of vibration of stretched string-using sonometer

Recommended Texts:

1. Halliday, D., Resnick, R. & Walker, J. (2014). *Fundamentals of physics* (10th Ed.). New York: Wiley.
2. Halliday, D., Resnick, R. & Krane, K. S. (2003). *Physics* (5th Ed.). New York: Wiley.

Suggested Readings:

1. Young, H. D., Freedman, R. A. & Ford, A. L. (2019). *University physics* (15th Ed.). New York: Pearson.
2. Serway, R. A. & Jewett, J. W. (2014). *Physics for scientists and engineers* (9th Ed.). New York: Brooks/Cole.



Machine learning is one of the fastest growing areas of computer science, with far-reaching applications. The aim of this course is to:

- a) Present the basic machine learning concepts.
 - b) Present a range of machine learning algorithms along with their strengths and weaknesses.
- Apply machine learning algorithms to solve problems of moderate complexity.

Contents

1. Introduction to machine learning
2. concept learning: General-to-specific ordering of hypotheses
3. Version spaces Algorithm, Candidate elimination algorithm
4. Supervised Learning: decision trees, Naive Bayes
5. Artificial Neural Networks, Support Vector Machines
6. Overfitting, noisy data, and pruning, Measuring Classifier Accuracy
7. Linear and Logistic regression
8. Unsupervised Learning: Hierarchical Agglomerative Clustering
9. k-means partitional clustering; Self-Organizing Maps (SOM);
10. k-Nearest-neighbor algorithm
11. Semi- supervised learning with EM using labeled and unlabeled data
12. Reinforcement Learning: Hidden Markov models
13. Monte Carlo inference Exploration vs. Exploitation Trade-off
14. Markov Decision Processes
15. Ensemble Learning: Using committees of multiple hypotheses
16. Bagging, boosting

Recommended Texts:

1. Kelleher, J. D. (2019). Machine learning: The basics. MIT Press.
2. Alpaydin, E. (2021). Introduction to machine learning (4th ed.). MIT Press.

Suggested Readings:

1. Mitchell, T. M. (1997). Machine learning. McGraw-Hill.
2. Murphy, K. P. (2012). Machine learning: A probabilistic perspective. MIT Press.



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This course use to introduce BS students to the basic concepts, principles and practices of scientific writing and research method in the social sciences. The course is designed to equip students with the skills necessary to communicate scientific information clearly and effectively. It also equips with the knowledge and skills necessary to design, conduct, analyse, and report empirical research. The course covers quantitative methodologies, research design, sampling techniques, data collection methods, and ethical considerations.

Contents

Unit 1: Foundations of Scientific Writing

1. Introduction to Scientific Writing
2. Structure of Scientific Papers (IMRaD Format)
3. Types of Scientific Documents (articles, reviews, theses, etc.)

Unit 2: The Nature of Educational Research

4. Definitions of Research, scope and importance
5. Research problem and topic
6. Characteristics of research problem

Unit 3: Variables and Hypotheses/Research Questions

7. Variable and Types of Variables
8. Hypotheses and Research Questions
9. Types of hypotheses and Testing the Hypothesis

Unit 4: Research Methodology

9. Research Domains: Qualitative, Quantitative, and Mix-method
10. Types of Research Designs: Descriptive, Correlation, Experimental
11. Population and Sampling Techniques
12. Development and Validation of Research Tool
13. Data Analysis: Descriptive and Inferential

Unit 5: Research Proposal and Report Writing

14. Writing formats
15. Proper Referencing and Citation Styles (APA, MLA, Chicago, etc.)
16. Ethics in Scientific Writing (Plagiarism, Authorship, Data Fabrication)

Recommended Text

1. Garg, R. (2024). *Research Methodology and Scientific Writing* (2nd ed.). Springer.
<https://doi.org/10.1007/978-3-030-64865-7>

Suggested Readings

1. John, W. C. (2018). *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research*, (6th Ed.), New York: Pearson Education. Retrieved from http://www.sxf.uevora.pt/wp-content/uploads/2013/03/Creswell_2012.pdf



The fundamental goal of this course is to create understanding in students to classical mechanics and its applications. The focus in this course will be given to develop knowledge of the physical concepts and mathematical methods of classical mechanics to improve skills in formulating and solving physics problems. Students will learn the use of Newton's laws of motion, conservation theorems to solve advanced problems involving the dynamic motion of classical mechanical systems.

Contents

1. Newtonian Mechanics-Single Particle: Newton's Laws
2. Frame of Reference, The Equation of motion for a particle
3. Conservation theorems, Energy
4. Limitations of Newtonian Mechanics
5. Oscillations: Simple Harmonic Oscillator
6. Harmonic Oscillations in Two dimensions
7. Damped Oscillations, Sinusoidal Driving Force
8. Physical Systems.
9. Gravitation: Gravitation Potential, Lines of Force and Equipotential Surfaces
10. Ocean Tides
11. Central Force Motion: Conservation Theorems, Equations of Motion
12. Orbits in a central Field, Centrifugal Energy and Effective Potential
13. Planetary Motion-Kepler's Problem, Orbital Dynamics
14. Dynamics of System of Particles: Center of mass
15. Linear momentum of system
16. Angular momentum of system

Recommended Texts:

1. Thornton, S. T. & Marion, J. B. (2012). *Classical dynamics of particles and systems* (5th ed.). New York: Thomson Brooks/Cole
2. Tai L. Chow. (2010). *Classical mechanics* (2nd ed). Taylor and Francis. California USA
3. Goldstein, H., Charles, P. P. & Safko J. L. (2001). *Classical mechanics* (3rd ed). Massachusetts: Addison Wesley Reading.

Suggested Readings:

1. Taylor, J. R. (2005). *Classical mechanics*. California: University Science Books.



List of Deficiency Courses

Calculus is the mathematical study of continuous change. If quantities are continually changing, we need calculus to study what is going on. Calculus is concerned with comparing quantities which vary in a non-linear way. It is used extensively in science & engineering, since many of the things we are studying (like velocity, acceleration, current in a circuit) do not behave in a simple, linear fashion. Calculus has two major branches, differential calculus (Calculus-I) & integral calculus (Calculus-II); the former concerns instantaneous rates of change, & the slopes of curves, while integral calculus concerns accumulation of quantities, & areas under or between curves. This is the first course of the sequence, Calculus-I, II & III, serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts & skills needed for mathematical manipulation. It focuses on the study of functions of a single variable. Calculus-I is an introduction to differential & integral calculus: the study of change.

Contents

- 1 Functions: Functions & their graphs, combining functions, Shifting and Scaling Graphs, Trigonometric functions, Exponential function, Inverse function and logarithmic, Rates of change & tangents to curves.
- 2 Limit and Continuity: Limit of a function & limit laws, the precise definition of a limit One-sided limits, continuity, Limits involving infinity; asymptotes of graphs
- 3 Derivatives: tangents Lines & derivative at a point, the derivative as a function Differentiation rules, the derivative as a rate of change, Derivatives of trigonometric functions, Chain rule, implicit differentiation, Derivative of inverse functions and logarithms, inverse trigonometric functions. Related rates, linearization & differentials, higher derivatives
- 4 Applications of derivatives: extreme values of functions, Rolle's theorem, the mean value theorem, Monotonic functions & the first derivative test, Convexity, point of inflection & second derivative test, Concavity & curve sketching, Indeterminate forms & L'Hôpital's rule, Applied optimization, Antiderivatives,
- 5 Integrals: area & estimating with finite sums, sigma notation & limits of finite sums, definite integral, the fundamental theorem of calculus, Indefinite integrals & the definite integral and the substitution and area between curves

Recommended Texts

1. Thomas, G.B., Weir, M. D., & Hass J. R. (2014). *Thomas' calculus: single variable* (13th ed./Latest). London: Pearson.
2. Stewart, J. (2015). *Calculus* (8th ed. /Latest). Boston: Cengage Learning.

Suggested Readings

1. Anton, H., Bivens I. C., & Davis, S. (2016). *Calculus* (11th ed. /Latest). New York: Wiley.
2. Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed.). London: Pearson.
3. Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed. /Latest). New York: Brooks Cole.

This is the second course of the basic sequence Calculus serving as the foundation of advanced subjects in all areas of mathematics. The sequence, equally, emphasizes basic concepts & skills needed for mathematical manipulation. As continuation of Calculus-I, it focuses on the study of functions of a single variable. This Core Curriculum course is designed to meet the following four learning goals: Students will construct and evaluate logical arguments. Students will apply and adapt a variety of appropriate strategies to solve mathematical problems. Students will recognize and apply mathematics in contexts outside of mathematics. Students will organize and consolidate mathematical thinking through written and oral communication. Students will integrate transcendental functions, including logarithms, exponential, trigonometry and inverse trigonometric, hyperbolic and inverse hyperbolic functions, apply methods of integration, such as algebraic substitution, trigonometric substitution, partial fractions, integration by parts, and use a table of integrals, solve limit problems involving indeterminate forms with La'Hopital's Rule and convert parametric representation of curves to rectangular coordinates, represent a curve using polar coordinates, and integrate functions expressed in polar coordinates.

Contents

- 1 Applications of definite integrals: volumes using cross-sections, Volumes using cylindrical shells, arc length, Areas of surfaces of revolution
- 2 Transcendental functions: Inverse functions & their derivatives, Natural logarithms, exponential functions, Inverse trigonometric functions, hyperbolic functions, related rates of Growth.
- 3 Techniques of integration:
Using Basic Integration Formulas, Integration by Parts, Trigonometric Integrals, Trigonometric Substitutions, Integration of Rational Functions by Partial Fractions, Integral Tables & Computer Algebra Systems, Numerical Integration, Improper Integrals
- 4 Infinite Sequences and Series:
Sequences, Infinite Series, The Integral Test, Comparison Tests, Absolute Convergence, The Ratio & Root Tests, Alternating Series & Conditional Convergence, Power Series, Taylor & Maclaurin Series, Convergence of Taylor Series, The Binomial Series & Applications of Taylor Series
- 5 Parametric equations and Polar Coordinates:
Parametrizations of Plane Curves, Calculus with Parametric Curves, Polar Coordinates, Graphing Polar Coordinate Equations, Areas & Lengths in Polar Coordinates, Conic Sections, Conics in Polar Coordinates.

Pre-requisite: Calculus-I

Recommended Texts

- 1 Thomas, G. B., Weir, M. D., & Hass, J. R. (2014). *Thomas' calculus: single variable* (13th ed. /Latest). London: Pearson.
- 2 Stewart, J. (2012). *Calculus*, (8th ed. /Latest). New York: Cengage Learning.

Suggested Readings

- 1 Anton, H., Bivens, I. C., & Davis, S. (2016). *Calculus*, (11th ed. /Latest). New York: Wiley.
- 2 Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed.). London: Pearson.
- 3 Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed. /Latest). New York: Brooks Cole.

This is an introductory course in discrete mathematics. Discrete Mathematics is study of distinct, un-related topics of mathematics; it embraces topics from early stages of mathematical development & recent additions to the discipline as well. It is the study of mathematical structures that are fundamentally discrete rather than continuous. In contrast to real numbers that have the property of varying "smoothly", the objects studied in discrete mathematics, such as integers, graphs, & statements in logic. The goal of this course is to introduce students to ideas and techniques from discrete mathematics that are widely used in science and engineering. This course teaches the students techniques in how to think logically and mathematically and apply these techniques in solving problems. To achieve this goal, students will learn logic and proof, sets, functions, as well as algorithms and mathematical reasoning. Key topics involving relations, graphs, trees, and formal languages and computability are covered in this course. The present course restricts only to counting methods, relations & graphs. The objective of the course is to inculcate in the students the skills that are necessary for decision making in non-continuous situations.

Contents

- 1 Counting methods: Basic methods: product
- 2 Inclusion-exclusion formulae
- 3 Mathematical Induction
- 4 Permutations & combinations
- 5 Recurrence relations & their solutions
- 6 Generating functions
- 7 Double counting & its applications
- 8 Pigeonhole principle & its applications
- 9 Relations: Binary relations, n-ary Relations, closures of relations
- 10 Composition of relations, inverse relation
- 11 Graphs: Graph terminology
- 12 Representation of graphs
- 13 Graphs isomorphism
- 14 Algebraic methods: the incidence matrix, connectivity
- 15 Eulerian & Hamiltonian paths, shortest path problem
- 16 Trees & spanning trees, Complete graphs & bivalent graphs

Recommended Texts

1. Rosen, K.H. (2012). *Discrete mathematics & its applications*. New York: The McGraw-Hill Companies, Inc.
2. Chartr, G., & Zhang, P. (2012). *A first course in graph theory*. New York: Dover Publications, Inc.

Suggested Readings

1. Tucker, A. (2002). *Applied combinatorics*. New York: John Wiley & Sons.
2. Diestel, R. (2010). *Graph theory* (4th ed.). New York: Springer- Verlag
3. Brigs, N. L. (2003). *Discrete mathematics*. Oxford: Oxford University Press.

This is the third course of the basic sequence Calculus-I, II & III, serving as the foundation of advanced subjects in all areas of mathematics. It focuses on the study of functions of a multivariable. The main focus of the course is to the study of multiple integrals in different coordinate systems & their applications. Moreover, a brief introduction to vector calculus will also be presented.

Contents

- 1 Vectors and the geometry of space:
Three-dimensional Coordinate System, Vectors, The dot product, The cross product, Lines & planes in space, Cylinder & Quadric surfaces,
- 2 Vector valued functions and Motion in space:
Curves in space and their tangents, Derivatives & integrals of vector functions, Arc length & Curvature, Motion in space, Velocity & Acceleration, Tangential & Normal Components of Acceleration, Velocity & Acceleration in Polar Coordinates
- 3 Functions of several variables: limits, Continuity & partial derivatives, The Chain rule, directional derivatives & the gradient vector, Maximum & minimum values, optimization problems, Lagrange Multipliers
- 4 Multiple integrals: Double integrals over rectangles & iterated integrals, Double integrals over general regions, Double integrals in polar coordinates, Triple integrals in rectangular, cylindrical & spherical coordinates, Applications of double & triple integrals, Change of variables in multiple integrals
- 5 Vector calculus: Vector fields, line integrals, The fundamental theorem of Line Integrals
Green's theorem, Curl & divergence, Surface integrals over scalar & vector fields
Divergence theorem, Stokes' theorem

Pre-requisite: Calculus-II

Recommended Texts

1. Thomas, G. B., Weir, M.D., & Hass J.R. (2014). *Thomas' Calculus: multivariable* (13th ed. /Latest). London: Pearson.
2. Stewart, J. (2015). *Calculus* (8th ed. /Latest). New York: Cengage Learning.

Suggested Readings

1. Anton, H., Bivens, I. C., & Davis, S. (2016). *Calculus* (11th ed. /Latest). New York: Wiley.
2. Goldstein, L. J., Lay, D. C., Schneider, D. I., & Asmar, N. H. (2017). *Calculus & its applications* (14th ed. /Latest). London: Pearson.
3. Larson, R., & Edwards, B. H. (2013). *Calculus* (10th ed. /Latest). New York: Brooks Cole.

This course is an introduction to group theory, one of the three main branches of pure mathematics. Group theory is the study of groups. Group theory is one of the great simplifying and unifying ideas in modern mathematics. It was introduced in order to understand the solutions to polynomial equations, but only in the last one hundred years has its full significance, as a mathematical formulation of symmetry, been understood. It plays a role in our understanding of fundamental particles, the structure of crystal lattices and the geometry of molecules. In this course, we will begin by defining the axioms satisfied by groups and begin to develop basic group theory by reference to some elementary examples. We will analyze the structure of 'small' finite groups, and examine examples arising as groups of permutations of a set, symmetries of regular polygons and regular solids, and groups of matrices. We will develop the notions of homomorphism, normal subgroups and quotient groups and study the First Isomorphism Theorem and its application.

Contents

- 1 Groups, definition & examples of groups, elementary properties of groups
- 2 Finite & Infinite Groups, Order of element of a group & related results
- 3 Subgroups, examples of subgroup, subgroup tests, subgroup generated by set
- 4 Cyclic groups, properties of cyclic groups
- 5 Classification of subgroups of cyclic groups
- 6 Cosets decomposition of a group, properties of cosets
- 7 Conjugate elements & conjugacy classes, Lagrange's theorem & its consequences
- 8 Centralizer of a subset of a group, normalizer of a subset of a group
- 9 Center of group definition & examples
- 10 Normal Subgroups, factor groups, application of factor groups
- 11 Permutations & Permutation groups, definition & examples
- 12 Homomorphism of groups, properties of Homomorphisms
- 13 Fundamental theorem of homomorphism
- 14 Isomorphism theorems, properties of Isomorphisms & Cayley's theorem
- 15 Endomorphism & automorphisms of groups, Commutator subgroups
- 16 External & Internal direct products, definition & examples
- 17 Rings: Definition, examples
- 18 Examples of non-commutative rings, Polynomial rings
- 19 Matrix rings. Units, zero-divisors
- 20 Nilpotent, idempotents. Subrings, Ideals, Maximal & prime Ideals.

Recommended Texts

1. Gallian, J.A. (2017). *Contemporary abstract algebra* (9th ed.). New York: Brooks/Cole.
2. Malik, D. S., Mordeson, J. N. & Sen, M.K. (1997). *Fundamentals of abstract algebra*. New York: WCB/McGraw-Hill.

Suggested Readings

1. Roman, S. (2012). *Fundamentals of group theory* (1st ed.). Basel: Birkhäuser.
2. Rose, H. E. (2006). *A course on finite groups* (1st ed.). London: Springer-Verlag.
3. Fraleigh, J.B. (2003). *A first course in abstract algebra* (7th ed.). Boston: Addison-Wesley Publishing Company.

This course is designed primarily for those students taking courses in mathematics. Vector and tensor algebra have in recent years become basic part of fundamental mathematical background required of those in engineering, sciences and allied disciplines. It is said that vector and tensor analysis is a natural aid in forming mental pictures of physical and geometrical ideas. A most rewarding language and mode of thought for the physical sciences. The focus, therefore, is to impart useful skills on the students in order to enhance their Mathematical ability in applying vector technique to solve problems in applied sciences and to equip them with necessary skill required to cope with higher levels courses in related subjects. Topics to be covered in this course include, basic vector algebra, coordinate bases, gradient, divergence, and curl, Green's, Gauss' and Stokes' theorems. The metric tensor, Christoffel symbols and Riemann curvature tensor. Applications will be drawn from differential geometry, continuum mechanics, electromagnetism, general relativity theory.

Contents

- 1 Vector Analysis: Scalar triple product with applications
- 2 Vector triple product with applications
- 3 Gradient of a scalar function
- 4 Divergence of vector functions
- 5 Curl of vector functions
- 6 Application of the del operator
- 7 Curvilinear coordinates
- 8 Coordinates surfaces
- 9 Cartesian Tensors: Summation convention
- 10 Transformation equations
- 11 Orthogonally conditions
- 12 Kronecker delta & Levi-civita symbol
- 13 Tensors of different ranks
- 14 Symmetric & anti symmetric tensors
- 15 Related theorems
- 16 Application to Vector Analysis

Recommended Texts

1. Shah, N.A. (2015). *Vector & tensor analysis*. Lahore: Ilmi Ketab Khana.
2. Spiegel, M.R. (2016). *Vector & Introduction to tensor analysis*. New York: McGraw Hill.
3. Yousuf, S.M. (1988). *Elementary Vector analysis*. Lahore: Ilmi Ketab Khana.

Suggested Readings

1. Young, E.C. (1993). *Vector & tensor analysis*. New York: Marcel Dekker, Inc.
2. Brand, L. (2006). *Vector analysis*, New York: Dover Publications.

Linear algebra is the study of linear systems of equations, vector spaces, and linear transformations. Solving systems of linear equations is a basic tool of many mathematical procedures used for solving problems in science and engineering. Linear Algebra plays a significant role in many areas of mathematics, statistics, engineering, the natural sciences, and the computer sciences. It provides a foundation of important mathematical ideas that will help students be successful in future coursework. The main objective of this course is to help students to learn in rigorous manner, the tools & methods essential for studying the solution spaces of problems in mathematics and in other fields & develop mathematical skills needed to apply these to the problems arising within their field of study and to various real-world problems. The student will become competent in solving linear equations, performing matrix algebra, calculating determinants, finding eigenvalues & eigenvectors and the student will come to understand a matrix as a linear transformation relative to a basis of a vector space.

Contents

- 1 Representation of linear equations in matrix form
- 2 Solution of linear system, Gauss-Jordan & Gaussian elimination method
- 3 Vector space, definition, examples & properties
- 4 Subspaces, Linear combination & spanning set
- 5 Linearly Dependent & Linearly Independent sets
- 6 Bases & dimension of a vector space
- 7 Intersections, sums & direct sums of subspaces, Quotient Spaces, Change of basis
- 8 Linear transformation, Rank & Nullity of linear transformation
- 9 Matrix of linear transformations
- 10 Eigen values & eigen vectors, Dual spaces
- 11 Inner product Spaces with properties, Projection
- 12 Cauchy inequality
- 13 Orthogonal & orthonormal basis
- 14 Gram Schmidt process & diagonalization

Recommended Texts

1. Dar, K.H. (2007). *Linear algebra* (1st ed.). Karachi: The Carwan Book House.
2. Kolman, B., & Hill, D. R. (2005). *Introductory linear algebra* (8th ed.). London: Pearson/Prentice Hall.

Suggested Readings

1. Cherney, D., Denton, T., Thomas, R., & Waldron, A. (2013). *Linear algebra* (1st ed.). California: Davis.
2. Anton, H., & Rorres, C. (2014). *Elementary linear algebra: applications version* (11th ed.). New York: John Wiley & Sons.
3. Grossman, S. I. (2004). *Elementary linear algebra* (5th ed.). New York: Cengage Learning.

This course shall assume background in calculus. This course introduces the fundamental principles in mechanics. Structural design applications of a variety of problems are developed throughout the course using examples that elucidate the theory of mechanics. It emphasizes on the laws of friction, equilibrium, center of gravity & harmonic & orbital motion. The objectives of the course are to develop better understanding of key concepts concerning scalar and vector fields learned previously in Multivariable Calculus courses, to gain deeper knowledge of multivariate differentiation operations such as Gradient, Divergent and Curl, master the Integral Theorems at the core of Vector Analysis: the Stokes (Greens') Theorem and the Divergence (Gauss') Theorem and to learn the utility of Vector Analysis by learning its relevance to Maxwell's equations describing the dynamics of electric and magnetic fields. In this course, students are prepared for further study in the relevant technological disciplines and more advanced mathematics courses.

Contents

- 1 Mechanics: Composition & resolution of co-planar forces, Moments
- 2 Couples & conditions of equilibrium under the action of co-planar forces
- 3 Frictional forces, Laws of friction
- 4 Equilibrium of bodies on rough surfaces
- 5 Principle of virtual work & related problems
- 6 Center of gravity, Center of mass of various bodies
- 7 Kinematics of a particle in Cartesian & polar co-ordinates
- 8 Linear & angular velocity
- 9 Rectilinear motion with uniform & variable acceleration
- 10 Simple harmonic motion
- 11 Projectile motion

Recommended Texts

1. Munawar, H., Saecd, S.M., & Ahmed, C.B. (2016). *Elementary vector analysis*. Lahore: The Caravan Book House.
2. Ghorī, Q.K. (2015). *Mechanics*. West Pakistan Publishing Company, Lahore:

Suggested Readings

1. Spiegel, M. R., Lipschutz, S., & Spellman, D. (2009). *Schaum's outline vector analysis* (2nd ed.). New York: McGraw-Hill Education.
2. Brand, L. (2006). *Vector analysis*. New York: Dover Publications.
3. Yousuf, S.M. (1988). *Vector analysis*. Lahore: Ilmi Ketab Khana.

Number theory (or arithmetic or higher arithmetic in older usage) is a branch of pure mathematics devoted primarily to the study of the integers & integer-valued functions. Integers can be considered either in themselves or as solutions to equations (Diophantine geometry). There are two subfields of number theory. One is Analytical Number Theory and other is Algebraic number theory. The focus of the course is on study of the fundamental properties of integers & develops ability to prove basic theorems. The specific objectives include study of division algorithm, prime numbers & their distributions, Diophantine equations & the theory of congruences. Students will learn about the arithmetic of algebraic number fields. They will learn to prove theorems about integral bases, & about unique factorization into ideals. They will learn to calculate class numbers, & to use the theory to solve simple Diophantine equations.

Contents

- 1 Divisibility
- 2 Euclid's theorem
- 3 Linear Diophantine Equation
- 4 Greatest common divisor and least common multiple
- 5 Congruences, Elementary properties
- 6 Residue classes & Euler's function
- 7 Linear congruence & congruence of higher degree
- 8 The theorems of Fermat
- 9 Euler & Wilson theorem
- 10 Cryptology, character Ciphers
- 11 Primitive roots & indices
- 12 Integers belonging to a given exponent
- 13 Quadratic Residues
- 14 Legendre symbol
- 15 Law of quadratic reciprocity, The Jacobi symbol
- 16 Number-Theoretic Functions
- 17 Mobius function

Recommended Texts

1. Rosen, K.H. (2000). *Elementary number theory & its applications*. (4th ed.). Boston: Addison-Wesley.
2. Apostol, T.M. (2010). *Introduction to analytic number theory* (3rd ed.). New York: Springer.

Suggested Readings

1. Leveque, W.J. (2002). *Topics in number theory*, Volumes I & II. New York: Dover Books.
2. Burton, D.M. (2007). *Elementary number theory*. New York: McGraw-Hill.

This course introduces the theory, solution, & application of ordinary differential equations. Topics discussed in the course include methods of solving first-order differential equations, existence & uniqueness theorems, second-order linear equations, power series solutions, higher-order linear equations, systems of equations, non-linear equations, Sturm-Liouville theory, & applications. The relationship between differential equations & linear algebra is emphasized in this course. An introduction to numerical solutions is also provided. Applications of differential equations in physics, engineering, biology, & economics are presented. The goal of this course is to provide the student with an understanding of the solutions & applications of ordinary differential equations. The course serves as an introduction to both nonlinear differential equations & provides a prerequisite for further study in those areas.

Contents

- 1 Introduction to differential equations: Preliminaries & classification of differential equations
- 2 Verification of solution, existence of unique solutions, introduction to initial value problems
- 3 Basic concepts, formation & solution of first order ordinary differential equations
- 4 Separable equations, linear equations, integrating factors, Exact Equations
- 5 Solution of nonlinear first order differential equations by substitution, Homogeneous Equations,
- 6 Bernoulli equation, Riccati's equation & Clairaut equation
- 7 Modeling with first-order ODEs: Linear models, Nonlinear models
- 8 Initial value & boundary value problems
- 9 Homogeneous & non-homogeneous linear higher order ODEs & their solutions, Wronskian,
- 10 Reduction of order, homogeneous equations with constant coefficients,
- 11 Nonhomogeneous equations, undetermined coefficients method, Superposition principle
- 12 Annihilator approach, variation of parameters, Cauchy-Euler equation,
- 13 Solving system of linear differential equations by elimination
- 14 Solution of nonlinear differential equations

Recommended Texts

- 1 Boyce, W. E., & DiPrima, R. C. (2012). *Elementary differential equations & boundary value problems* (10th ed.) USA: John Wiley & Sons.
- 2 Zill, D.G., & Michael, R. (2009) *Differential equations with boundary-value problems* (5th ed.) New York: Brooks/Cole.

Suggested Readings

- 1 Arnold, V. I. (1991). *Ordinary differential equations* (3rd ed.). New York: Springer.
- 2 Apostol, T. (1969). *Multi variable calculus & linear algebra* (2nd ed.). New York: John Wiley & sons.