UNIVERSITY OF SARGODHA
OFFICE OF THE REGISTRAR
(ACAD BRANCH)

NOTIFICATION

On the recommendations of Academic Council made in its 22nd (3/2024) meeting held on 30.09.2024, the Syndicate in its 69th (1/2025) meeting held on 17.01.2025 has approved the revised curricula of the following academic programs for implementation w.e.f Spring 2025:

1. M.Phil in Mathematics **(Annex-'A')**
2. Ph.D in Mathematics **(Annex-'B')**

(WAQAR AHMAD)
Additional Registrar (General)

No. SU/Acad/25/ 332

Dated: 13.03.2025

Distribution:

- Chairman, Department of Mathematics
- Controller of Examinations
- Director Academics

C.C:

- Dean, Faculty of Sciences
- Director, QEC
- Additional Registrar (Affiliation & Registration)
- Secretary to the Vice-Chancellor
- PA to Registrar
- Notification File

Ref. no. SU/Math/210
February 26-2025

Annex - 'A'

SCHEME OF STUDIES

Master of Philosophy in Mathematics

(w.e.f. Spring 2025)

DEPARTMENT OF MATHEMATICS
UNIVERSITY OF SARGODHA
SARGODHA

1. **Nomenclature of the Program:**

Master of Philosophy in Mathematics

2. **Department Brief:**

History of the department is as old as the institution itself. Postgraduate classes, however, were introduced in 1985. The department has been producing outstanding experts for teaching at intermediate, degree and postgraduate level who earned remarkable repute at national and international levels. The graduates of the department are serving in government, semi-government and private sectors of both educational and non-educational departments. The curricula have been revised as per initiatives taken by the HEC and the feedback of the faculty. The department of mathematics is an important pillar of academia in almost every university/institute which is awarding degrees in sciences and engineering. The department consists of 18 faculty members of in which 16 are PhDs with specializations in core and advanced areas of mathematics. Currently the department offers BS-4 years, BS-5th Intake 2 years, MPhil (Self-support and weekend) and PhD programs with specialization in various fields of mathematics. A total of 1100 students from all over Pakistan are enrolled in these programs. We welcome you to the Department of Mathematics under Faculty of Sciences. Through our degree programs, we aim to provide the highest level of quality education in the field of Pure, Applied and Computational Mathematics along with current development in mathematical research. To nurture research culture, we organize national and international conferences annually by inviting mathematicians from across the globe. Moreover, the Department has established several educational collaborations, both national and international, to expedite the current research trends. We guide, teach, and support students of all mathematical abilities and interests. We welcome students into our community of mathematical scholars. We share our ideas, grow in our knowledge, and learn from one another. We discover new mathematics using creativity and ingenuity.

3. **Program Learning Objectives:**

The curriculum for M.Phil students is designed by the collaboration of mathematicians and engineers. Students undertaking research in this department will have a chance to learn not only the fundamental courses of mathematics but also advanced courses related to their area of specialization and interest. Emerging specializations in the domain of mathematics, like pure mathematics, applied mathematics and computational mathematics, will be offered as

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area of research for graduate students at this department. It is also working as supporting department for other sciences, social sciences and engineering and department so upon successful completion of the courses taught by mathematics faculty, students will be able to:

- develop a deep understanding of advanced mathematical concepts, theories, and techniques across various branches of mathematics such as pure, applied, and computational,
- acquire proficiency in conducting independent and original research in mathematics, including formulating research questions, designing methodologies, and analyzing results,
- cultivate the ability to critically evaluate mathematical problems, conjectures, and proofs, and to devise innovative solutions using rigorous logical reasoning and creative problem-solving techniques,
- enhance oral and written communication skills to effectively communicate complex mathematical ideas, results, and proofs to both specialist and non-specialist audiences,
- explore the application of mathematical principles and techniques to real-world problems in various fields such as physics, engineering, computer science, finance, and biology etc,
- engage in interdisciplinary collaboration and exchange ideas with researchers from other disciplines to leverage mathematical insights in addressing interdisciplinary challenges,
- develop skills in teaching undergraduate and graduate-level mathematics courses, mentoring students, and effectively communicating mathematical concepts in an educational setting,
- prepare for diverse career paths in academia, industry, government, and non-profit sectors by acquiring transferrable skills such as project management, problem-solving, teamwork, and leadership.

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4. Program Structure:

Duration	Minimum 2-Years (4-Semesters), Maximum 4-Years (8-Semesters)
Entry Requirements:	Candidates having minimum 2nd division in annual system or CGPA 2.0 out of 4.00 in MA/M.Sc/BS (4-Years)/ BS 5th-Intake (2-Years)/B.Sc (Hons.)/ BS (4-Years) in any specialization etc. degree (16 years of education) in semester system/Annual system in Mathematics subject from HEC recognized Institutions+ Departmental admission test (qualifying 50% marks).
Degree Completion Requirements:	Total Credit Hours of Course Work: 24 Total Credit Hours of Thesis: 06 Total Credit Hours of Program: 30
Program Mode (select one)	Thesis Track
Specialization (if any)	N/A

5. List of Mandatory/Compulsory/Core Courses:

Sr. No.	Course Code	Course Title	Credit Hours	Prerequisite
1.	MATH-7101	Partial Differential Equations with Applications	3(3-0)	Nil
2.	MATH-7102	Numerical Solutions of Ordinary Differential Equation	3(3-0)	Nil
3.	MATH-7103	Mathematical Techniques for Boundary Value Problems	3(3-0)	Nil
4.	MATH-7104	Research Methodology	3(3-0)	Nil
5.	MATH-7105	Integral Transform	3(3-0)	Nil
6.	MATH-7106	Advanced Numerical Analysis	3(3-0)	Nil
7.	MATH-7107	Fractional Calculus	3(3-0)	Nil
8.	MATH-7108	Mathematical Computing	3(3-0)	Nil
9.	MATH-7109	Inequalities Involving Convex Functions	3(3-0)	Nil
10.	MATH-7110	Generalized Special Functions	3(3-0)	Nil
11.	MATH-7111	Interpolating Spline Functions	3(3-0)	Nil
12.	MATH-7112	Computer Aided Geometric Design	3(3-0)	Nil
13.	MATH-7113	Advanced Fluid Mechanics	3(3-0)	Nil
14.	MATH-7114	Riemannian Geometry	3(3-0)	Nil
15.	MATH-7115	Semigroup Theory	3(3-0)	Nil
16.	MATH-7116	Commutative Semigroup Rings	3(3-0)	Nil

6. List of Elective Courses:

Sr. No.	Course Code	Course Title	Credit Hours	Prerequisite
1.	MATH-7117	Representation Theory-I	3(3-0)	Nil
2.	MATH-7118	Representation Theory-II	3(3-0)	MATH-7117
3.	MATH-7119	Advanced Ring Theory-I	3(3-0)	Nil
4.	MATH-7120	Advanced Ring Theory-II	3(3-0)	MATH-7119
5.	MATH-7121	Fixed Point Theory	3(3-0)	Nil
6.	MATH-7122	Topological Algebras	3(3-0)	Nil

7.	MATH-7123	Commutative Algebra-I	3(3-0)	Nil
8.	MATH-7124	Commutative Algebra-II	3(3-0)	MATH-7123
9.	MATH-7125	Convex Analysis	3(3-0)	Nil
10.	MATH-7126	Theory of Group Actions	3(3-0)	Nil
11.	MATH-7127	Theory of Group Graphs	3(3-0)	Nil
12.	MATH-7128	Advanced Complex Analysis-I	3(3-0)	Nil
13.	MATH-7129	Advanced Complex Analysis-II	3(3-0)	MATH-7128
14.	MATH-7130	Variational Inequalities	3(3-0)	Nil
15.	MATH-7131	Field Extensions and Galois Theory	3(3-0)	Nil
16.	MATH-7132	Harmonic Analysis	3(3-0)	Nil
17.	MATH-7133	Dynamic Inequalities on Time Scales	3(3-0)	Nil
18.	MATH-7134	Set-Valued Analysis	3(3-0)	Nil
19.	MATH-7135	Fuzzy Algebra	3(3-0)	Nil
20.	MATH-7136	Advanced Fuzzy Set Theory	3(3-0)	Nil
21.	MATH-7137	Magnetohydrodynamics-I	3(3-0)	Nil
22.	MATH-7138	Magnetohydrodynamics-II	3(3-0)	MATH-7137
23.	MATH-7139	Advanced Analytical Dynamics-I	3(3-0)	Nil
24.	MATH-7140	Advanced Analytical Dynamics-II	3(3-0)	MATH-7139
25.	MATH-7141	General Relativity	3(3-0)	Nil
26.	MATH-7142	Elastodynamics-I	3(3-0)	Nil
27.	MATH-7143	Elastodynamics-II	3(3-0)	MATH-7142
28.	MATH-7144	Advanced Heat Transfer	3(3-0)	Nil
29.	MATH-7145	Cosmology	3(3-0)	Nil
30.	MATH-7146	Electrodynamics-I	3(3-0)	Nil
31.	MATH-7147	Electrodynamics-II	3(3-0)	MATH-7146
32.	MATH-7148	Perturbation Methods-I	3(3-0)	Nil
33.	MATH-7149	Perturbation Methods-II	3(3-0)	MATH-7148
34.	MATH-7150	Viscous Fluids-I	3(3-0)	Nil
35.	MATH-7151	Viscous Fluids-II	3(3-0)	MATH-7150
36.	MATH-7152	Environmental Heat Transfer	3(3-0)	Nil
37.	MATH-7153	Graph Theory	3(3-0)	Nil
38.	MATH-7154	Approximation Theory	3(3-0)	Nil
39.	MATH-7155	Introduction to Subdivision Scheme	3(3-0)	Nil
40.	MATH-7156	Design Theory	3(3-0)	Nil
41.	MATH-7157	Acoustics	3(3-0)	Nil
42.	MATH-7158	Combinatorics	3(3-0)	Nil
43.	MATH-7159	Theory of Majorization	3(3-0)	Nil
44.	MATH-7160	Nilpotent and Solvable Groups	3(3-0)	Nil
45.	MATH-7161	Mathematical Modeling-I	3(3-0)	Nil
46.	MATH-7162	Mathematical Modeling-II	3(3-0)	MATH-7161
47.	MATH-7163	Computer Graphics	3(3-0)	Nil
48.	MATH-7164	Minimal Surfaces	3(3-0)	Nil
49.	MATH-7165	Soliton Wave Theory	3(3-0)	Nil
50.	MATH-7166	Algorithm Analysis	3(3-0)	Nil
51.	MATH-7167	Advanced Mathematical Physics	3(3-0)	Nil
52.	MATH-7168	Advanced Spline Functions	3(3-0)	Nil
53.	MATH-7169	Theory of Soft Sets and Hybrid Structures	3(3-0)	Nil

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7. **Thesis:**

1.	MATH-7199	**Thesis**	6	

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Scheme of Studies

Master of Philosophy in **MATHEMATICS**

Semester-I

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Compulsory-1	MATH-71xx	To be selected from List*	3(3-0)	Nil
Compulsory-2	MATH-71xx	To be selected from List*	3(3-0)	Nil
Elective-1	MATH-71xx	To be selected from List**	3(3-0)	Nil
Elective-2	MATH-71xx	To be selected from List**	3(3-0)	Nil

Semester-II

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Compulsory-3	MATH-71xx	To be selected from List*	3(3-0)	Nil
Compulsory-4	MATH-71xx	To be selected from List*	3(3-0)	Nil
Elective-3	MATH-71xx	To be selected from List**	3(3-0)	Nil
Elective-4	MATH-71xx	To be selected from List**	3(3-0)	Nil

Semester-III-IV

Category	Course Code	Course Title	Credit Hours	Pre-Requisite
Compulsory	MATH-7199	Thesis	06	

Prram Summary:

Category	Minimum No of Courses	Minimum No of Credit Hours
Compulsory Courses	04	12
Elective Courses	04	12
Thesis	---	06

* These are four compulsory courses and they can be selected from the list of mandatory/compulsory/core courses as per availability of the resources.

** These are four elective courses and they can be selected from the list of elective courses as per availability of the resources. Student from MPhil may opt elective courses from the list of PhD elective courses if being offered in PhD class in same semester and same as, student from PhD may opt elective courses from the list of MPhil elective courses if being offered in MPhil class in same semester, after approval from departmental post graduate committee.

LIST OF COURSES

MATH -7101 **Partial Differential Equations with Applications** **3(3+0)**

The course objectives are to learn the basics analytical methods to solve partial differential equations (PDE). Partial Differential Equations (PDEs) are at the heart of applied mathematics and many other scientific disciplines. The theory of partial differential equations (PDE) is important in pure and applied mathematics. On the one hand this is used for mathematical formulae in many phenomena from the natural sciences (electromagnetism, Maxwell's equations) or social sciences (financial markets, Black-Scholes model). On the other hand since the pioneering work on surfaces and manifolds by Gauss and Riemann partial differential equations have been at the centre of many important developments on other areas of mathematics (geometry, Poincare-conjecture). Subject of the module are four significant partial differential equations (PDEs) which feature as basic components in many applications: The transport equation, the wave equation, the heat equation, and the Laplace equation. This will discuss the qualitative behavior of solutions and, thus, be able to classify the most important partial differential equations into elliptic, parabolic, and hyperbolic type. Possible initial and boundary conditions and their impact on the solutions will be investigated. Solution techniques comprise the method of characteristics, Green's functions, and Fourier series. The well-posed boundary value problems in sense of Dirichlet, Neumann and Robin conditions are very useful in the classical solutions of linear as well as nonlinear PDEs applicable to many fields in engineering and technology. Applications include problems from fluid dynamics, electrical and mechanical engineering, materials science and quantum mechanics.

Contents

1. Cauchy's problems for linear second order equations in n-independent variables
2. Cauchy Kowalewski Theorem
3. Characteristics surfaces
4. Adjoint operations
5. Bicharacteristics Spherical and Cylindrical
6. Waves. Heatequation
7. Waveequation
8. Laplace equation
9. Maximum-Minimum Principle
10. Integral Transforms

Recommended Texts

1. Lawrence, C. E. (2010). *Partial differential equations* (2nd ed.). United States: American Mathematical Society
2. David, L. J. (2004). *Applied partial differential equations* (2nd ed.). New York: Springer-Verlag

Suggested Readings

1. Tynmyint, U. & Lokenath, D. (2006). *Linear partial differential equations for scientists and engineers* (4th ed.). United States: Birkhäuser Boston
2. Jürgen, J. (2002). *Partial differential equations.* New York: Springer
3. Recent research articles.

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MATH-7102 **Numerical Solutions of Ordinary Differential Equation** **3 (3+0)**

The main focus of this course is to get understand for the numerical solutions of the formulated problems based upon ordinary derivatives. In mathematics, an ordinary differential equation (ODE) is a differential equation containing one or more functions of one independent variable and the derivatives of those functions. The term ordinary is used in contrast with the term partial differential equation which may be with respect to more than one independent variable. Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals. *Numerical Methods* for *Ordinary Differential Equations* is a self-contained introduction to a fundamental field of *numerical* analysis and scientific computation. Many differential equations cannot be solved using symbolic computation. For practical purposes, however, such as in engineering, a numeric approximation to the solution is often sufficient. The algorithms studied here can be used to compute such an approximation. An alternative method is to use techniques from calculus to obtain a series expansion of the solution. Ordinary differential equations occur in many scientific disciplines in physics, chemistry, biology, and economics. In addition, some methods in numerical partial differential equations convert the partial differential equation into an ordinary differential equation, which must then be solved. The course also provides hands-on experience on implementing numerical algorithms for solving engineering problems using Mathematica and MATLAB software.

Contents

1 Theory and implementation of numerical methods for initial and boundary value problems in ordinary differential equations
2 One-step, linear multi-step, Runge-Kutta of two, Runge-Kutta of three, Runge-Kutta of four
3 Extrapolation methods, Convergence, stability
4 Error estimates, Practical implementation, Study and analysis of shooting
5 Finite difference and projection methods for boundary value problems for ordinary differential equation

Recommended Texts

1 Donald, G. (2008). *Numerical solution of ordinary differential equations*. Germany: Wiley-VCH
2 Butcher, J. C. (2016). *Numerical methods for ordinary differential equations* (3rd ed.). United Kingdom: Wiley

Suggested Readings

1 Lawrence, F. S. (1994). *Numerical solution of ordinary differential equation.* New York: Chapman & Hall Mathematics CRC Press
2 Shampine, L. F., Gladwell, I. & Thompson, S. (2003). *Solving ODEs with MATLAB.* Cambridge: Cambridge University Press
3 Kendall, A., Weimin, H. & David, E. S. (2009). *Numerical solution of ordinary differential equations.* New Jersey: Wiley
4 Recent published paper

MATH -7103 **Mathematical Techniques for Boundary Value Problems** **3 (3+0)**

This course has a major focus on training analytical, logical thinking and learning fundamental methods for solving ordinary and partial differential equations. Both the knowledge about differential equations as well as the training of analytical faculties will be useful for the students in the course of their further studies. In mathematics, in the field of differential equations, a boundary value problem is a differential equation together with a set of additional constraints, called the boundary conditions. A solution to a boundary value problem is a solution to the differential equation which also satisfies the boundary conditions. Boundary value problems arise in several branches of physics as any physical differential equation will have them. Problems involving the wave equation, such as the determination of normal modes, are often stated as boundary value problems. A large class of important boundary value problems is the Sturm–Liouville problems. The analysis of these problems involves the Eigen functions of a differential operator. To be useful in applications, a boundary value problem should be well posed. This means that given the input to the problem there exist a unique solution, which depends continuously on the input. Much theoretical work in the field of partial differential equations is devoted to proving that boundary value problems arising from scientific and engineering applications are in fact well-posed. Among the earliest boundary value problems to be studied is the Dirichlet problem, of finding the harmonic functions (solutions to Laplace's equation); the solution was given by the Dirichlet's principle.

Contents

1 Green's function method with applications to wave-propagation
2 Keller Box scheme
3 Local Non-Similarity method
4 Shooting method
5 A survey of transform techniques
6 Wiener-Hopf technique with applications to diffraction problems

Recommended Texts

1 Stakgold, I. (1987). *Boundary value problems of mathematical physics.* Philadelphia: Society for Industrial and Applied Mathematics
2 Na, T. Y. (2012). *Computational methods in engineering boundary value problems.* New York: Academic Press

Suggested Readings

1 Noble, B. (1998). *Methods based on the wiener-hopf technique for the solution of partial differential equations* (2nd ed.). New York: American Mathematical Society
2 Recent research articles

MATH-7104 **Research Methodology** **3 (3+0)**

The purpose of research is to discover answers to questions through the application of scientific procedures. The main aim of research is to find out the truth which is hidden and which has not been discovered as yet. The students will gain familiarity with a phenomenon or to achieve new insights into it (studies with this object in view are termed as exploratory or formulate research studies). The students should be able to identify the overall process of designing a research study from its inception to its report.

Contents

1 Scientific statements, hypothesis, model
2 Theory & Law, Types of research, Problem definition
3 Objectives of the research, research design, data collection
4 Data analysis, Interpretation of results, validation of results
5 Limitation of Science, calibration, Sensitivity
6 Least count and reproducibility, Stability and objectivity
7 Difference between accuracy and precision
8 Literature search, defining problem, Feasibility study
9 Pilot projects / field trials, Formal research proposal
10 Budgeting and funding, Progress report, final technical and fiscal report
11 Purpose of experiment, good and bad experiments
12 Inefficient experiments, null and alternative hypothesis
13 Alpha and beta errors, Relationship of alpha and beta errors to sensitivity and specificity
14 Designing efficient experiments, Simple random sampling
15 Systematic sampling, Stratified sampling, cluster sampling
16 Convenience sampling, judgment sampling, quota sampling
17 Snow ball sampling , Identifying variables of interest and their interactions
18 Operating characteristic curves, power curves, Surveys and field trials
19 Submission of a paper, role of editor, Peer-review process
20 Importance of citations, impact factor, Plagiarism
21 Protection of your work from misuse, Simulation
22 Need for simulation, types of simulation
23 Introduction to algorithmic research, algorithmic research problems
24 Types of algorithmic research, problems, types of solution procedure

Recommended Texts

1 Kumar, R. (2010). *Research methodology: A step-by-step guide for beginners* (3rd ed.). New York: Sage Publications Ltd.
2 Harre, R. (2002). *Great scientific experiments: Twenty experiments that changed the world*: New York: Dover Pub.

Suggested Readings

1 Day, R.A. (1979). *How to write and publish a scientific paper*. USA: ISI Press, Philadelphia.
2 Diamond, W. J. (1989). *Practical experiment designs for scientists and engineers* (2nd ed.). New York: John Wiley.

MATH-7105 **Integral Transform** **3 (3+0)**

The goals for the course are to gain a facility with using the transform, both specific techniques and general principles, and learning to recognize when, why, and how it is used. Together with a great variety, the subject also has a great coherence, and the hope is students come to appreciate both. The Students will be able to know the use of Laplace transform in system modeling, digital signal processing, process control, solving Boundary Value Problems. Students will be able to use Fourier transform in communication theory and signal analysis, image processing and filters, data processing and analysis, solving partial differential equations for problems on gravity. The students will be able to use Z-transform in the characterization of Linear Time Invariant system (LTI), in development of scientific simulation algorithms. The student will be able to use the Mellin transform to solve various problems. The students will be able to solve different problems by using the Hankel transform. In mathematics, an integral transform maps an equation from its original domain into another domain where it might be manipulated and solved much more easily than in the original domain. The solution is then mapped back to the original domain using the inverse of the integral transform.

Contents

1 Laplace transform
2 Application to integral equations
3 Fourier transforms
4 Fourier sine and cosine transform
5 Inverse transform, application to differentiation
6 Convolutions theorem, application to partial differential equations
7 Hankel transform and its applications
8 Application to integration
9 Mellin transform and its applications
10 Abel transform
11 Hilbert transform
12 Jacobi transform

Recommended Texts

1 Davies, B. (2002). *Integral transforms and their applications* (3rd ed.). New York: Springer.
2 Trigub, R.M. & Belinsky, E.S. (2010). *Fourier analysis and approximation of functions*. New York: Springer.

Suggested Readings

1 Pinkus, A. & Zafrany, S. (1997). *Fourier series and integral transforms*. Cambridge: Cambridge University Press.
2 Vasistha, R. & Gupta, R.K. (2007). *Integral transform*. India: Krisna Prakashan Media Pvt. Ltd.
3 Recent research articles.

MATH-7106 **Advanced Numerical Analysis** **3 (3+0)**

Advanced numerical analysis is essential in making numerical weather prediction feasible. Computing the trajectory of a spacecraft requires the accurate measuring in the target achievements. This course is the continuation of Numerical Analysis. The student will learn state-of-the-art algorithms for solving ordinary differential equations, nonlinear systems, and optimization problems. Moreover, the analysis of these algorithms and their efficient implementation will be discussed in some detail. The emphasis will be on both the analysis and the implementation of these methods. The same idea for the approximation of the solution by a series expansion (truncated) will be used for the numerical computations. The student will learn some basic theoretical results on approximations for the issues of stability and convergence, on practical algorithms for implementing numerical methods, and on designing efficient and accurate spectral algorithms for solving mathematical problems. More advanced techniques for numerical computations will be introduced for the better learning of MATLAB skills in *numerical* methods, programming and graphics. After this, one can learn easily with Scilab for *advanced numerical analysis* package similar to MATLAB or Octave for a complete GUI and Xcos which is alternative to Simulink frequently used in weather forecasting in super computers for advanced technologies.

Contents

1 Introduction, Euler's method
2 The improved and modified Euler's method
3 Runge-Kutta method, Milnes method
4 Hammign's methods, Initial value problem
5 The special cases when the first derivative is missing
6 Boundary value problems
7 The simultaneous algebraic equations method
8 Iterative methods for linear equations
9 Gauss-Siedel method, Relaxation methods
10 Vector and matrix norms, Sequences and series of matrices
11 Graph Theory, Directed graph of a matrix
12 Strongly connected and irreducible matrices
13 Grerschgoin theorem, Symmetric and positive definite matrices
14 Cyclic-Consistently ordered matrices
15 Choice of optimum value for relaxation parameter

Recommended Texts

1 Richard, L. B. & Faires, J. D. (2010). *Numerical analysis* (9th ed.). New York: Brooks Cole
2 Stanislaw, R. (2008). *Fundamental numerical methods for electrical engineering. Lecture notes in electrical engineering*. Berlin: Springer

Suggested Readings

1 Gerald, C. F. & Wheatley, P. O. (1994). *Applied numerical analysis*. Boston: Addison-Wesley Publishing Company
2 Recent research articles.

MATH-7107 **Fractional Calculus** **3 (3+0)**

The subject of fractional calculus came to life over the last few decades. A feature is that engineers and scientists have developed new models that involve fractional differential equations. These models have been applied successfully, e.g., in mechanics (theory of viscoelasticity), bio-chemistry (modelling of polymers and proteins), electrical engineering (transmission of ultrasound waves), medicine (modelling of human tissue under mechanical loads), etc. The mathematical theory seems to be lagging the needs of those applications, but the wealth of applications indeed indicates the truth of the above quote from Heaviside. There are some aspects that can be summarized as the "pure mathematical" side of the problems without taking into consideration those questions that arise in the applications mentioned above, and some that the engineer's point of view without a rigorous mathematical justification of the ideas. This subject attempts to fill the gap between these two approaches. There established mathematically a sound theory of the differential equations that have been shown to be relevant in practice and provide a thorough mathematical analysis. The fundamentals of fractional calculus with its applications is also a goal of this course to provide a solid foundation that may later be used for the construction of efficient and reliable numerical methods for fractional differential equations. A successful development and a thorough understanding of such numerical schemes is not possible without such a stable analytical background. The students are assumed to be familiar with classical calculus (differential and integral calculus and the elementary theory of differential equations) to cope this advanced course in the context of theory of Lebesgue integrals.

Contents

1 Introduction; motivation
2 Basics
3 Application of fractional calculus
4 Riemann–Liouville
5 Differential and integral operators
6 Riemann–Liouville integrals, Riemann–Liouville
7 Derivatives, relations between Riemann–Liouville integrals and derivatives
8 Grunwald–Letnikov operators
9 Caputo's approach
10 Definition and basic properties
11 Non-classical representations of Caputo operators

Recommended Texts

1 Diethelm, K. (2010). *The analysis of fractional differential* equations, New York: (Springer-Verlag Berlin Heidelberg).
2 Kilbas, A. A., Srivastava, H. M. & Trujillo, J. J. (2006). *Theory and applications of fractional differential equations.* North-Holland Mathematics Studies: Elsevier.

Suggested Readings

1 Oldham, K. B. & Spanier, J. (2006). *The fractional calculus: Theory and applications of differentiation and integration to arbitrary order*. New York: Dover Books on Mathematics.
2 Related research papers.

MATH-7108 **Mathematical Computing** **3(3+0)**

Programming Languages plays an important role in Mathematics. A number of computer software available to deal with mathematical computing and simulation. This course provides a practical introduction to most widely used Mathematical computing software's namely, Mathematica or Mat Lab. After this course students will be able to develop computer programs in these software according to their requirements in mathematical computing.

Contents

Mathematica
1 Introduction to the basic environment of Mathematica and its syntax
2 Running Mathematica
3 Numerical/Algebraic Calculations
4 Vectors, Matrices, Sets, Lists, Tables, arrays
5 Symbolic Mathematics in Mathematica
6 Functions and functional programming
7 Procedural programming
8 Do, for and while loops
9 Flow controls
10 Graphics, Plots of 2D and 3D functions
11 Packages within Mathematica

Mat Lab
1. Introductory Demonstration of Mat Lab
2. Symbolic computations in Mat Lab
3. Vectors, Matrices, Sets, Lists, Tables, arrays and Arrays
4. Toolbars and Palettes
5. Operators, Constant, Elementary Functions, Procedures
6. If clauses, selection and conditional execution
7. Looping, for and while loop, looping commands
8. Recursion
9. Plots of 2D and 3D functions, Packages within Mat Lab

Recommended Texts

1. Wellin P., Kamin S. Gaylord R. (2011). *An introduction to programming with Mathematica*, (3rd ed.). Cambridge: Cambridge University Press.
2. Hunt, B. R., Lipsman, R. L., & Rosenberg, J. M. (2014). *A guide to MATLAB: for beginners and experienced users*. Cambridge: Cambridge University Press.
3. Lipsman, R., & Rosenberg, J. M. (2001). *A Guide to MATLAB*. Cambridge: Cambridge University Press.

Suggested Readings

1. Maeder, R. E. (1997). *Programming in Mathematica* (3rd ed.). Boston: Addision-Weseley.
2. Hoste, J. (2009). *Mathematica demystified*. New York: McGraw Hill.
3. Wolfram, S. (2003). *The Mathematica book*. Wolfram Research, Inc.

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MATH-7109 **Inequalities involving Convex Functions** **3 (3+0)**

The iinequalities play an important role in almost all branches of mathematics as well as in other areas of science. The basic work "Inequalities" by Hardy, Littlewood and Pólya appeared in 1934 and the books "Inequalities" by Beckenbach and Bellman published in 1961 and "Analytic Inequalities" by Mitrinovi´c published in 1970 made considerable contributions to this field and supplied motivations, ideas, techniques and applications. Since 1934 an enormous amount of effort has been devoted to the discovery of new types of inequalities and to the application of inequalities in many parts of analysis. The usefulness of mathematical inequalities is felt from the very beginning and is now widely acknowledged as one of the major driving forces behind the development of modern real analysis. The theory of inequalities is in a process of continuous development state and inequalities have become very effective and powerful tools for studying a wide range of problems in various branches of mathematics. This theory in recent years has attracted the attention of many researchers, stimulated new research directions, and influenced various aspects of mathematical analysis and applications. Among the many types of inequalities, those associated with the names of Jensen, Hadamard and Hermite have deep roots and made a great impact on various branches of mathematics. The last few decades have witnessed important advances related to these inequalities that remain active areas of research and have grown into substantial fields of research with many important applications. The development of the theory related to these inequalities resulted in a renewal of interest in the field and has attracted interest from many researchers. A host of new results have appeared in the literature. The present course provides a systematic study of some of the most famous and fundamental inequalities originated by the above-mentioned mathematicians and brings together the latest, interesting developments in this important research area under a unified framework.

Contents

1. Jensen's and related inequalities
2. Some general inequalities involving convex functions
3. Hadamard's inequalities
4. Inequalities of Hadamard type I
5. Inequalities of Hadamard type II
6. Some inequalities involving concave functions
7. Miscellaneous inequalities

Recommended Texts

1 Pachpatte, B. G. (2005). *Mathematical inequalities*. New York: Elsevier.
2 Pečarić, J., Proschan, F. and Tong, Y. C. (1992). *Convex functions, partial orderings and statistical applications* (Vol. 187). Cambridge: Academic Press, Boston, Mass.

Suggested Readings

1. Mitrinovic, D. S., J.Pečarić and Fink, A. M. (1993). *Classical and new inequalities in analysis*. Netherlands: Kluwer Academic Publishers.
2. Recent research articles.

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MATH-7110 **Generalized Special Functions** **3 (3+0)**

The main aim of this course is the study of the properties and relations of special functions such as incomplete gamma, beta functions, zeta, hypergeometric, confluent hypergeometric functions, Bessel functions and generalized hypergeometric functions. Furthermore, the properties, relations and applications of these special functions are also discussed in the form a new direction k>0. The students will able to find some results and applications associated with special k-functions. Special functions are those mathematical functions which more or less have established their identities due to their undoubted usefulness in mathematical analysis, functional analysis, physics, chemistry, and other fields of science, technology and industry. Many special functions appear as solutions of differential equations or integrals of elementary functions. Therefore, tables of integrals usually include descriptions of special functions, and tables of special functions include most important integrals; at least, the integral representation of special functions. Because symmetries of differential equations are essential to both physics and mathematics, the theory of special functions is closely related to the theory of Lie groups and Lie algebras, as well as certain topics in mathematical physics.

Contents

1 Infinite products
2 Properties and applications of special functions
3 The incomplete gamma, beta functions
4 Zeta functions
5 The hypergeometric functions and identities
6 Generalized hypergeometric functions
7 Bessel functions
8 The confluent hypergeometric functions
9 Introduction to q-series, k- hypergeometric functions
10 Generalized k-hypergeometric functions
11 Confluent k-hypergeometric functions

Recommended Texts

1 Richard, B. (2016). *Special functions and orthogonal polynomials*. Cambridge: Cambridge University Press.
2 Rainville, E.D. (1971). *Special functions* (3rd ed.). New York: The Macmillan Company.

Suggested Readings

1 Andrews, G. E., Richard, A. & Roy, R. (2000). *Special functions* (1st ed.).Cambridge: Cambridge University Press.
2 Mathai, A.M. & Houbold, H.J. (2008). *Special functions for applied scientists*. New York: Springer Science and Business Media, LLC.
3 Singh, U.P. and Denis, R.Y. (2001). *Special functions and their applications*. New Dehli: Dominant Publishers and Distributors.
4 Recent research articles.

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MATH-7111 **Interpolating Spline Functions** **3(3-0)**

In the mathematical field of numerical analysis, spline interpolation is a form of interpolation where the interpolant is a special type of piecewise polynomial called a spline. That is, instead of fitting a single, high-degree polynomial to all of the values at once, spline interpolation fits low-degree polynomials to small subsets of the values, for example, fitting nine cubic polynomials between each of the pairs of ten points, instead of fitting a single degree-nine polynomial to all of them. Spline interpolation is often preferred over polynomial interpolation because the interpolation error can be made small even when using low-degree polynomials for the spline. Spline interpolation also avoids the problem of Runge's phenomenon, in which oscillation can occur between points when interpolating using high-degree polynomials. The course aims to present an introduction to splines interpolation. After successful completion, students should be able to design, implement & use interpolations for computer solution of scientific problems involving problems generated by set of data.

Contents

1 Interpolatory cubic splines,
2 The representation of s in terms of the values Mi=s(2)(xi), i=0,1,2,...,k.,
3 The representation of s in terms of the values mi=s(1)(xi), i=0,1,2,...,k.,
4 Cubic and Quadratic Hermite spline,
5 Theorems regarding error analysis,
6 Theorems regarding to Convergence of the D1, D2,
7 Natural and periodic splines,
8 End conditions for cubic Hermite spline interpolation,
9 E(α)-cubic splines.

Recommended Texts

1 Farin, G. (2002). *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*. Academic Press Inc.
2 David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science +Business Media Inc.
3 Bartels, R.H., Bealty, J.C. and Beatty, J.C. (2006). *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*. Morgan Kaufmann Publisher.
4 Faux, I.D. (1979). *Computational Geometry for Design and Manufacture*. Ellis Horwood.

Suggested Readings

1 de Boor, C. (2001). *A Practical Guide to Splines*. UK. Springer Verlag.
2 Schumaker, L.L. (1993). *Spline Functions: Basic Theory*. John Wiley.
3 Wang, R.H. (2005). *Multivariate Spline Functions and Their Applications (Mathematics and its Applications)*. Science Press/ Kluwer Academic Publishers.
4 Bartels, R.H., Bealty, J.C. and Beatty, J.C. (2006). *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*. Morgan Kaufmann Publisher.
5 Recent Published papers

MATH-7112 **Computer Aided Geometric Design** **3(3+0)**

The primary goals are to (1) learn about different parametric curve and surface schemes, (2) understand the advantages and disadvantages of different geometry representations, (3) gain practical experience by implementing several CAGD techniques and user interfaces and (4) to apply CAGD methods to practical applications. Interactive graphics techniques for defining and manipulating geometrical shapes used in computer animation, car body design, aircraft design, and architectural design. In this course follow a modular approach and contribute different components to the development of an interactive curve and surface modeling system. There are two Modeling Techniques: (1) Curve Modeling Techniques have outcomes: Students will implement various curve interpolation and approximation techniques that allow the interactive specification of three-dimensional curves (e.g. Bezier, B-spline, rational curves). (2) Surface modeling techniques have outcomes: Students will implement various surface interpolation and approximation techniques that allow the interactive specification of three-dimensional surfaces (e.g. Bezier, B-spline, rational surfaces). Simple, 3D Modeling System: Students will integrate the curve and surface modules into a system that allows the user to interactively design and store simple, 3D geometries.

Contents

1 Linear interpolation, Piecewise linear interpolation blossoms, Barycentric coordinates in the plane, The de Casteljau algorithm, Properties of Bezier curves, Bernstein polynomials
2 Composite Bezier curves, Degree elevation, The variation diminishing property
3 Degree reduction, Polynomial curve constructions
4 Aitken's Algorithm, Lagrange Polynomials, Lagrange interpolation, Cubic Hermite interpolation
5 Point-normal interpolation, B-Spline curves, B-spline segments, Knot insertion, degree elevation
6 Greville Abscissae, Smoothness, Constructing Splines Curves, modifying B-Spline curves
7 Cubic spline interpolation, the minimum property
8 Piecewise cubic interpolation.
9 Rational Bezier and B-Spline Curves,
10 Rational Cubic B-spline curves.

Recommended Texts

1 Gerald F. (2002), *Curves and surfaces for CAGD, a practical guide (5th ed.)*. Massachusetts: Morgan Kaufmann Publishers.
2 Bartels, R. H., Bealty, J. C. and Beatty, J. C. (2006). *An Introduction to spline for use in computer graphics and geometric modeling*. Massachusetts: Morgan Kaufmann Publishers.
3 David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science +Business Media Inc.

Suggested Readings

1 Josef H., Dieter L. (1993). *Fundamentals of computer aided geometric design*. Massachusetts: A K Peter, Ltd.
2 de Boor, C. (2001). *A practical guide to splines*. New York: Springer Verlag
3 Wang, R. H. (2005). *Multivariate spline functions and their applications (mathematics and its applications)*. Netherland: Science Press/ Kluwer Academic Publishers.
4 Recent Published papers

MATH -7113 **Advanced Fluid Mechanics** 3(3+0)

The main purpose of this course is not so much to feed students with "advanced" material (the topics covered do not in fact appear terribly advanced). It is instead designed to help students develop a mastery of the underlying principles and the ability to solve, quickly and efficiently, a variety of real fluid mechanics problems from first principles. The lectures present and illustrate the fundamental laws and the methods and modeling approximations that form the basis of fluid mechanics. The problems and tutorials help the students gain mastery of the material and to develop, by practice and trial and error, the mindset of an effective problem solver in fluid mechanics. This course is a survey of principal concepts and methods of fluid dynamics. Topics include mass conservation, momentum, and energy equations for continua; Navier-Stokes equation for viscous flows; similarity and dimensional analysis; lubrication theory; boundary layers and separation; circulation and vorticity theorems; potential flow; introduction to turbulence; lift and drag; surface tension and surface tension driven flows.

Contents

1. Navier-Stoke's equation and exact solutions
2. Dimensional Analysis
3. Dynamical similarity and Reynold's number
4. Laminar flat plate boundary layer: exact solution, momentum, integral equation, use of momentum integral Equation for flow with zero pressure gradient
5. Turbulent flow
6. Boundary layer concept and governing equations
7. Pressure gradient in boundary-layer flow
8. Reynold's equations of turbulent motion
9. MHD equations
10. Fluid drifts
11. Stability and equilibrium problems

Recommended Texts

1. Rahman, M. and Brebbia, C. A. (2008). *Advances in fluid mechanics VII*. England: WIT Press.
2. Batchelor, G. K. (2000). *An introduction to fluid dynamics*. Cambridge: Cambridge University Press.

Suggested Readings

1. Francis F. C. (2010). *Introduction to plasma physics and controlled fusion*. New York: springer.
2. Krall, N. A., and Trivelpiece, A. W. (1986). *Principles of plasma physics*. San Francisco: San Francisco Press, Incorporated.
3. Recent research articles

MATH-7114 **Riemannian Geometry** **3 (3+0)**

Riemannian geometry is the branch of differential geometry that studies riemanian manifolds, smooth manifold with a riemanian metric that is with an inner product on the tangent space at each point that varies smoothly from point to point. Since Euclidian geometry is the study of flat space, between each point of point there is a unique line segment which is the shortest curve. This line segment can be extanded to lines of infinite length. This course with main purpose is to introduce the beautiful theory of Riemannian geometry, a still very active area of mathematical research. The study of Riemannian geometry is rather meaningless without some basic knowledge on Gaussian geometry i.e. the geometry of curves and surfaces in 3-dimensional Euclidean space.

Contents

1. Definitions and examples of manifolds
2. Tangents
3. Coordinate vector fields
4. Tangent spaces, Dual spaces
5. Algebra of tensors, Vector fields
6. Tensor fields, Integrals curves
7. Affine connections and Christoffel symbols
8. Covariant differentiation of tensor fields
9. Geodiscs equations
10. Curve on manifold
11. Parallel transport
12. Lie transport
13. Lie derivatives and Lie brackets
14. Geodisc deviation
15. Differential form
16. Introduction to integration theory on manifolds
17. Riemannian curvature tensor
18. Ricci tensor and Ricci scalar
19. Killing equations and killing vector fields

Recommended Texts

1. Bishop, R. L. and Goldberg S. I. (1980). *Tensor analysis on manifolds* (1st Ed.). New York: Dover Publication.
2. Carmo M. P. (1992). *Riemannian geometry* (1st Ed.). Boston Switzerland: Birkahauser.

Suggested Readings

1. Langwitz, D. (1970). *Differential and riemannain geometry*. Cambridge: Academic Press.
2. Abraham, R., Marsden, J. E., and Ratiu, T. (1993). *Manifolds tensor analysis and applications*, (Addison-Wesley).
3. Recent research articles.

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MATH-7115 **Semigroup Theory** **3(3+0)**

In some respects we can think of a semigroup as an abstraction of a group but on the other hand it is sometimes useful to compare the theory of semigroups with that of rings (the 'multiplicative part' of a ring is just a semigroup) and many of the historical developments in the theory of semigroups owe much to these two theories. However recent work has highlighted strong connections with, for example, many aspects of theoretical computer science (automata theory, theory of codes and formal language theory) as well as with other areas of mathematics such as the theory of ordered structures and (partial) symmetries.

Contents

1 Semigroups: Introductory ideas and basic definitions
2 Cyclic semigroups
3 Ordered sets
4 Semi lattices
5 Binary relations
6 Equivalences congruences
7 Free semi-groups
8 Green's Equivalences
9 L,R,H,J and D Regular semi groups
10 O-Simple semigroups
11 Simple and O-Simple semi groups
12 Rees's theorem
13 Primitive idempotent
14 O-Simple semi groups
15 Finite congruence free semigroups
16 Union of groups
17 Bands
18 Free bands varieties of bands Inverse semigroups
19 Congruences on Inverse semigroups
20 Fundamental inverse semigroups
21 Bisimple and simple inverse semigroups
22 Orthodox semigroups
23 Basic properties
24 The structure of orthodox semigroups

Recommended Texts

1 Howie J. M. (1976). *An introduction to semigroup theory.* Cambridge: Academic Press.
2 Clifford, A. H. and Preston. G. B. (1967). *The algebraic theory of semigroups.* Vol. I & II. Michigan: AMS Math. Surveys.

Suggested Readings

1 Howie J. M. (1976). *An introduction to semigroup theory.* Cambridge: Academic Press.
2 Related Research Papers.

MATH -7116 **Commutative Semigroup Rings** **3(3+0)**

The purpose of this course to understand the basic properties of commutative semigroup rings. Commutative Semigroup Rings was the first exposition of the basic properties of semigroup rings. This course concentrates on the interplay between semigroups and rings, thereby illuminating both of these important concepts in modern algebra. The course begins with the introduction to commutative rings and the basic notions of commutative semigroups. It continues with some of the main properties and results on cyclic semigroups and ordered semigroups, illustrated with many examples. After introducing the basic terminologies in semigroup ring, the course ends with the Monoid domains.

Contents

1 Commutative Rings: Definition and examples
2 Integral domains, unit, irreducible and prime elements in ring
3 Types of ideals, Quotient rings, Rings of fractions
4 Ring homomorphism, definitions and examples of Euclidean domains
5 Principle ideal domains and unique factorization domains
6 Definition and examples of DVRs, Dedkind and Krull domains
7 Commutative Semigroups, basic notions
8 Cyclic semigoups, Numerical Monoids
9 Ordered semigroups, congruences, Noetherian semigroups
10 Factorization in commutative Monoids
11 Semigroup Ring and its Distinguished Elements
12 Introduction of polynomial rings in one indeterminate
13 Structure of semigroup ring, Zero divisors
14 Nilpotent elements, idempotents, units
15 Ring theoretic properties of Monoid Domains
16 Integral dependence for domains and Monoid domains
17 Monoid domains as factorial domains, monoid domains as Krull domains
18 Divisor class group of a Krull Monoid domain

Recommended Texts

1 Chapman, S. T. (2005). *Arithmetical properties of commutative rings and monoids* (1st ed.). Florida: CRC Press.
2 Gilmer, R. (1972). *Multiplicative ideal theory* (1st ed.). New York: Marcel Dekker.

Suggested Readings

1 Matsumura, H. (1986). *Commutative ring theory* (1st ed.). Cambridge: Cambridge University Press.
2 Gilmer, R. (1984). *Commutative semigroup rings* (1st ed.). Chicago: The University of Chicago Press.

MATH-7117 **Representation Theory-I** 3(3+0)

The aim of the course is to give an introduction to the theory of representation. The chief emphasis will be on the three areas: finite groups, compact Lie groups and complex Lie algebras. Representation theory is a fundamental tool for studying symmetry by means of linear algebra: it is studied in a way in which a given group or algebra may act on vector spaces, giving rise to the notion of a representation. Representation theory is an area of mathematics which, roughly speaking, studies symmetry in linear spaces. It is a beautiful mathematical subject which has many applications, ranging from number theory and combinatorics to geometry, probability theory, quantum mechanics and quantum field theory. Representation theory was born in 1896 in the work of the German mathematician F. G. Frobenius. This work was triggered by a letter to Frobenius by R. Dedekind. In essence, a representation makes an abstract algebraic object more concrete by describing its elements by matrices and the algebraic operations in terms of matrix addition and matrix multiplication.

Contents

1 Preliminaries from group theory
2 Group representations
3 FG-modules
4 FG-sub-modules
5 Reducibility
6 Group algebras
7 FG-homeomorphisms
8 Maschke's Theorem
9 Schur's lemma
10 Irreducible modules and the group algebra
11 Conjugacy classes
12 Characters
13 Inner products of characters
14 The number of irreducible characters.

Recommended Texts

1 James, G., Liebech, M. (2001). *Representations and characters of groups* (2nd ed.). Cambridge: Cambridge University Press.
2 Tullio, C. S., Fabio, S., Filipoo, T. (2008). *Harmonic analysis on finite groups: Representation Theory, Gelfand Pairs and Markov Chains* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1 Charles, W. C., Irving, R. (2006). *Representation theory of finite groups and associative algebras* (1st ed.). Michigan: American Mathematical Society.
2 William, F., and Young, T. (2012). *Young tableaux: with applications to representation theory and geometry* (3rd ed.). Cambridge: Cambridge University Press.
3 Recent research articles.

MATH -7118 **Representation Theory-II** **3 (3+0)**

This course is continuation of the course Representation theory-I. The main topic of the course will be representation theory. The basic theory of linear representations of groups, with a particular focus on finite groups and representations defined over the complex numbers will be studied. We will also introduce the theory of characters as a tool for studying representations and we will develop techniques for constructing characters and character tables. We will also describe some important applications of the theory, including Burnside's famous theorem on the solubility of finite groups. We shall study the finite-dimensional representations of compact Lie groups from various points of view. The main objective will be the Weyl character formula. Then we'll turn to noncompact groups and infinite-dimensional representations. This is considerably more complicated, and at several places we shall limit ourselves to illustrating theorems in examples rather than proving them in general. This course also includes introduction to the Plancherel formula for complex groups and to the Langlands classification.

Contents

1 Character tables and orthogonality relations
2 Normal subgroups and lifted characters
3 Some elementary character tables
4 Tensor products
5 Restriction to a subgroup, induced modules and characters
6 Algebraic integers
7 Real representations
8 Summary of properties of character tables Characters of groups of order pq
9 Characters of some p-groups
10 Character tables of the sample group of order 168, Character table of GL (2,q)
11 Permutations and characters

Pre-requisition: Representation Theory-I

Recommended Texts

1 James, G., Liebech, M. (2001). *Representations and characters of groups* (2nd ed.). Cambridge: Cambridge University Press.
2 Tullio, C. S., Fabio, S., Filipoo, T. (2008). *Harmonic analysis on finite groups: Representation theory, gelfand pairs and markov chains* (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1 Charles, W. C., Irving, R. (2006). *Representation theory of finite groups and associative algebras* (1st ed.). Rhode Island: American Mathematical Society.
2 William, F., and Young, T. (2012). *Young tableaux: With applications to representation theory and geometry* (3rd ed.). Cambridge: Cambridge University Press.
3 Recent research articles.

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MATH-7119 **Advanced Ring Theory-I** **3(3+0)**

The philosophy of this subject is that we focus on similarities in arithmetic structure between sets (of numbers, matrices, functions or polynomials for example) which might look initially quite different but are connected by the property of being equipped with operations of addition and multiplication. The set of integers and the set of 2 by 2 matrices with real numbers as entries are examples of rings. These sets are obviously not the same, but they have some similarities and some differences - in terms of their algebraic structure. Although people have been studying specific examples of rings for thousands of years, the emergence of ring theory as a branch of mathematics in its own right is a very recent development. Much of the activity that led to the modern formulation of ring theory took place in the first half of the 20th century. Ring theory is powerful in terms of its scope and generality, but it can be simply described as the study of systems in which addition and multiplication are possible. A ring is an important fundamental concept in algebra and includes integers, polynomials and matrices. Ring theory has applications in number theory and geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. In this module we shall develop ring and module theory leading to the fundamental theorems of Wedderburn and some of its applications.

Contents

1 Radical classes
2 Semi-simple classes
3 The upper radical
4 Semi-simple images
5 The lower radical hereditariness of the lower radical class
6 The upper radical class
7 Partitions of simple rings

Recommended Texts

1 Huynh, D. V. (2010). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser.
2 López, P., Sergio, R., Van, H., Dinh. (2010). *Advances in ring theory*. Switzerland: Birkhäuser.
3

Suggested Readings

1 Wiegandt, R. (1974). *Radical and semisimple classes of rings* (1st ed.). Ontario: Queen's University.
2 Jain, S. K., Tariq, R. S. (1997). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser Basel.
3 Golan, J. S. (1999). *Semirings and their applications*. Netherlands: Springer
4 Recent research articles.

MATH -7120 **Advanced Ring Theory-II** **3 (3+0)**

This course is a continuation of the course Advance Ring Theory-I. A ring is an important fundamental concept in algebra and includes integers, polynomials and matrices as some of the basic examples. Ring theory has applications in number theory and geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring R provides us with an insight into the structure of R. In this module we shall develop ring and module theory leading to the fundamental theorems of Wedderburn and some of its applications. It develops the fundamental ideas of ring theory and is aimed at developing the student's ability to Radical, simple and semisimple artinian rings, the Artin-Wedderburn theorem and he concept of central simple algebras, the theorems of Wedderburn and Frobenius. Ring theory is powerful in terms of its scope and generality, but it can be simply described as the study of systems in which addition and multiplication are possible. A ring is an important fundamental concept in algebra and includes integers, polynomials and matrices. Ring theory has applications in number theory and geometry. A module over a ring is a generalization of vector space over a field. The study of modules over a ring *R* provides us with an insight into the structure of *R*. In this module we shall develop ring and module theory leading to the fundamental theorems of Wedderburn and some of its applications.

Contents

1 Minimal left ideals
2 Wedderburn-Artin structure theorem
3 The Brown-McCoy radical
4 the Jacobson radical
5 Connections among radical classes
6 Homomorphically closed semisimple classes

Pre-requisition: Advanced Ring Theory-I

Recommended Texts

1 Huynh, D. V. (2010). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser.
2 López, P., Sergio, R., Van, H., Dinh. (2010). *Advances in ring theory.* Switzerland: Birkhäuser.

Suggested Readings

1 Wiegandt, R. (1974). *Radical and semisimple classes of rings* (1st ed.). Ontario: Queen's University.
2 Jain, S. K., Tariq, R. S. (1997). *Advances in ring theory* (1st ed.). Switzerland: Birkhäuser Basel.
3 Golan, J. S. (1999). *Semirings and their applications*. Netherlands: Springer
4 Recent research articles

MATH-7121 **Fixed Point Theory** **3(3+0)**

The theory of fixed points has been revealed as a major, powerful and important tool in the study of nonlinear phenomena. The aim of this course is to understand the theory of fixed point which is an important branch of nonlinear functional analysis. This subject gives sufficient directions to Acquiring general knowledge in and concepts of fixed point theory as well as enabling students to successfully apply it when needed in other courses. This course is intended as a brief introduction to the subject with a focus on Banach Fixed Point theorems fixed point theorem and its application to nonlinear differential equations, nonlinear integral equations, real and complex implicit functions theorems and system of nonlinear equations. Some generalizations and similar results e. g. Kannan Fixed Point theorems, Banach Fixed Point theorem for multi-valued mappings are also educated.

Contents

1 Fixed point: Definitions and examples
2 Fixed point iteration procedure
3 Picard iteration
4 Banach's contraction principle
5 Contractive mappings on metric spaces and related fixed point theorems
6 Non-expansive mappings
7 Sequential approximation techniques for non-expansive mappings
8 Properties of fixed point sets and minimal set
9 Multivalued mappings and related fixed point theorems
10 The Topological fixed point property and Brouwer's fixed point theorem
11 Applications of fixed point theorems

Recommended Texts

1 Granas, A., Dugundji, J. (2010). *Fixed Point Theory* (1st ed.). New York: Springer.
2 Agarwal, R. P., Meehan, M., Regan, D. O. (2009). *Fixed Point Theory and Applications*, (1st ed.). Cambridge: Cambridge University Press.

Suggested Readings

1 Almezel, S., Ansari, Q. H., Khamsi, M. A. (2014). *Topics in fixed point theory*, (1st ed.). New York: Springer.
2 Berinde, V. (2007). *Iterative approximation of fixed points* (2nd ed.). New York: Springer-Verlag Berlin.
3 Goebel,K. and Kirk, W. A. (1990). *Topics in metric fixed point theory* (1st ed.). New York: Cambridge University Press.
4 Recent research articles.

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MATH -7122 **Topological Algebras** **3(3+0)**

The aim of this course is to study the topological spaces strongly related to groups: either the spaces themselves are groups in a nice way (so that all the maps coming from group theory are continuous), or groups act on topological spaces and can be thought of as consisting of homeomorphisms. Groups are attributed to Algebra. In the mathematics built on sets, main objects are sets with additional structure such as topology, metric, partial order. Topology and metric evolved from geometric considerations. Algebra studied algebraic operations with numbers and similar objects and introduced into the set-theoretic Mathematics various structures based on operations. One of the simplest (and most versatile) of these structures is the structure of a group. It emerges in an overwhelming majority of mathematical environments. It often appears together with topology and in a nice interaction with it. This interaction is a subject of Topological Algebra. In this course, we will study some important topological algebras including Q- algebras, Semi-simple algebra and Involutive algebra.

Contents

1 Definition of Topological algebra and its examples
2 Adjumetion of unity
3 Locally convex algebras. Idempotent and m-convex sets
4 Locally multicatively convex (l.m.c) algebras
5 Q-algebras
6 Frechet algebras, Spectrum of an element, Spectral radius
7 Basic theorems on Spectrum, Gelfand-Mazur theorem
8 Maximal ideals, quotient algebras
9 Multiplicative linear functional and their continuity
10 Gelfand transformations
11 Radical of an algebra
12 Semi-simple algebras
13 Involutive algebra
14 Gelfand-Naimark theorem l.m.c algebras

Recommended Texts

1 Arhangel'skii, A., Tkachenko, M. (2008). *Topological groups and related structures, an introduction to topological algebra* (1st ed.). Paris: Atlantis Press.
2 Mallios, A. (1986). *Topological algebras, selected topics*, (1st ed.). Hoofddrop: North-Holland Company.

Suggested Readings

1 Hussain, T. (1983). *Multiplicative functions on topological algebras, research notes in mathematics 85.* Boston: Pitman Advanced Publishing Program.
2 Beckenstein, E., Narici, L., uffel, C. (1977). *Topological algebras* (1st ed.). Hoofddrop: North-Holland Company.
3 Recent research articles.

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MATH -7123 **Commutative Algebra-I** **3(3+0)**

This course will be an introduction to commutative algebra, a subject that has interactions with algebraic geometry, number theory, combinatorics, and several complex variables. Commutative Algebra is the study of commutative rings, polynomial rings and their modules and ideals. This theory has developed over the last 150 years not just as an area of algebra considered for its own sake, but as a tool in the study of two enormously important branches of mathematics: algebraic geometry and algebraic number theory. The unification which results, where the same underlying algebraic structures arise both in geometry and in number theory, has been one of the crowning glories of twentieth century mathematics and still plays an absolutely fundamental role in current work in both these fields.

Contents

1. Commutative Rings: definition and examples
2. Integral domains, unit, irreducible and prime elements in ring
3. Types of ideals, Quotient rings, Rings of fractions, Ring homomorphism
4. Euclidean domains, principal ideal domains and unique factorization domains
5. Polynomial and Formal Power series Rings
6. Construction of formal power series ring R[[X]] and polynomial ring R[X] in one indeterminate,
7. Formal power series and polynomial rings in n indeterminate
8. Factorization in polynomial rings, irreducibility criteria,
9. Noetherian Rings: Definition and examples. Polynomial extension of Noetherian domains
10. Quotient ring of Noetherian rings, Ring of fractions of Noetherian rings,
11. Dimension of Rings: Chain of prime ideals in a domain, length of chain of prime ideals
12. Dimension of polynomial rings, Integral Dependence: Ring extension,
13. Integral element, almost integral element, integral closure of a domain
14. Complete integral closure of a domain, integrally closed and completely integrally closed domain,
15. Valuation Rings: definition and examples, Valuation map and value group, Rank of valuation
16. valuation map and valuation ring, valuation ring is integrally closed,
17. Discrete Valuation Rings and Dedekind domains: Fractional ideals,
18. Finitely generated fractional ideals, invertible fractional ideals, discrete valuation rings
19. Definitions and examples of Dedekind domains

Recommended Texts

1. Kemper, G. (2011). *A course in commutative algebra* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Atiyah, M. F., Macdonald, I. G. (1969). *Introduction to commutative algebra*, (1st ed.). Boston: Addison-Melbourne: Wesley Publishing Company.

Suggested Readings

1. Gilmer, R. (1972). *Multiplicative ideal theory* (1st ed.). New York: Marcell Dekker.
2. Matsumura, H. (1986). *Commutative Ring Theory* (1st ed.). Cambridge: Cambridge University Press.
3. Recent research articles.

MATH -7124 **Commutative Algebra-II** **3(3+0)**

This course is a continuation of the course Commutative algebra-I. Commutative algebra was born in the 19th century from algebraic geometry, invariant theory, and number theory. It has come to occupy a remarkably central role in modern mathematics, but in a different way. Today it is a mature field with activity on many fronts. The branch of mathematics which most of all draws upon commutative algebra for its structural integrity is algebraic geometry, algebraic number theory, and arithmetic geometry which is a field of mathematics that encompassed commutative algebra, classical algebraic geometry and algebraic number theory. The course aims at familiarizing the students with unique factorization domains, krull rings and factorial ring and finite factorization domains.

Contents

1. Unique Factorization Domains: Basics and examples
2. Guass Theorem, Quotient of a UFD, Nagata Theorem
3. Class Groups: Divisor classes, divisor class monoid, divisor class group
4. Krull Rings and Factorial Ring: Divisorial ideals
5. Divisors, Krull rings, stability properties, two classes of Krull rings
6. Divisor class groups, application of the theorem of Nagata, examples of factorial rings
7. Atomic Domains: Definition and examples, olynomial extension of Atomic domains
8. Domains Satisfying ACCP: Definition and examples
9. Polynomial extension of domains satisfying ACCP
10. Connection of domains satisfying ACCP and Atomic domains
11. Bounded Factorization Domains: Definition and examples.Length function
12. Characterization of BFD through length function.Polynomial extension of BFDs
13. Noetherian and Krull domains and BFDs, Half Factorial Domains: Class number of a Field
14. Carlitz Theorem, examples and basic results
15. Dedkind and Krull examples, inetegrabiltiy and HFD
16. On polynomial and polynomial like extensions
17. Finite Factorization Domains: Group of divisibility G(D) of a domain D
18. G(D) and FFD, Atomic idf-domain is FFD

Pre-requisition: Commutative Algebra-I

Recommended Texts

1. Kemper, G. (2011). *A course in commutative algebra* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Rotman, J. (2009). *An introduction to homological algebra* (2nd ed.). New York: Springer-Verlag.

Suggested Readings

1. Chapman, S. T., Glaz, S. (2000). *Non-Noetherian commutative ring theory* (1st ed.). New York: Springer.
2. Matsumura, H. (1986). *Commutative ring theory* (1st ed.). Cambridge: Cambridge University Press.
3. Recent research articles.

MATH-7125	Convex Analysis	3 (3+0)

Convex optimization problems are far more general than linear programming problems, but they share the desirable properties of LP problems: They can be solved quickly and reliably up to very large size hundreds of thousands of variables and constraints. The issue has been that, unless your objective and constraints were linear, it was difficult to determine whether they are convex. Convexity has been increasingly important in recent years in the study of extremum problems in many areas of applied mathematics. The purpose of this course is to provide an exposition of the theory of convex sets and functions in which applications to extremum problems play the central role. Systems of inequalities, the minimum or maximum of a convex function over a convex set, Lagrange multipliers, and minimax theorems are applications of convex analysis, along with the basic results about the structure of convex sets and the continuity and differentiability of convex functions and saddle-functions. A generalization of linear algebra is developed in which "convex bifunctions" are the analogues of linear transformations, and "inner products" of convex sets and functions are denned in terms of the extremal values in Fenchel's Duality Theorem. Each convex bifunction is associated with a generalized convex program, and an adjoint operation for bifunctions that leads to a theory of dual programs is introduced. The classical correspondence between linear transformations and bilinear functionals is extended to a correspondence between convex bifunctions and saddle-functions, and this is useful as the main tool in the analysis of saddle-functions and minimax problems.

Contents

1. Convex functions on the real line
2. Continuity and differentiability of convex functions
3. Characterizations, Differences of convex functions
4. Conjugate convex functions
5. Convex sets and affine sets
6. Convex functions on a normed linear space
7. Continuity of convex functions on normed linear space
8. Differentiable convex function on normed linear space
9. The support of convex functions
10. Differentiability of convex function on normed linear space

Recommended Texts

1. Niculescu, C. P. & Persson, L. E. (2018). *Convex functions and their applications, A contemporary approach* (2nd ed.). Canada: CMS Books in Mathematics.
2. Borwein, J. M. & Lewis, A. S. (2010). *Convex analysis and nonlinear optimization: Theory and examples* (2nd Ed.). New York: Springer.

Suggested Readings

1. Hiriart-Urruty, J. B. & Lemaréchal, C. (2004). *Fundamentals of convex analysis*. New York: Springer.
2. Roberts, A. W. & Varberg, D. E. (1973). *Convex functions*. New York: Academic Press.

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MATH -7126 **Theory of Group Actions** 3(3+0)

The purpose of this course is to give the introduction to theory of group actions. A group action is a representation of the elements of a group as symmetries of a set. the first group action studied was the action of the Galois group on the roots of a polynomial. However, there are numerous examples and applications of group actions in many branches of mathematics, including algebra, topology, geometry, number theory, and analysis, as well as the sciences, including chemistry and physics. Group theory is one of the great simplifying and unifying ideas in modern mathematics. It plays a role in our understanding of fundamental particles, the structure of crystal lattices and the geometry of molecules. In this course, we will begin by revising the simple axioms satisfied by groups and begin to develop basic group theory by reference to some elementary examples. We will also examine how the notion of a permutation group can be generalized to that of a group action on a set and will show how to use this in certain counting problems arising in combinatorics. We will also see how to use group actions to prove strong results about the structure of finite groups. Students will appreciate the value of abstraction and meet many examples of groups and group actions from around mathematics. Beyond theoretic aspects of group theory students will also see the value of these methods in the generality of the approach and also to otherwise intractable counting problems.

Contents

1 Preliminaries
2 The theory of group actions
3 Coset space
4 Multiplicative group of a finite fields
5 Extensions of finite fields
6 Projective line over finite fields
7 Projective
8 Linear groups through actions

Recommended Texts

1 Magnus, W., Karrass, A., Solitar, D. (2004). *Combinatorial group theory* (Revised ed.). New York: Dover Publications.
2 Rose, J. S. (1994). *A Course on group theory* (1st ed.). New York: Dover Publications.

Suggested Readings

1 Coxeter, H. S. M., Moser, W. O. J. (1980). *Generators and relations for discrete groups* (4th ed.). New York: Springer-Verlag Berlin Heidelberg.
2 Joseph, G. (2012). *Contemporary abstract algebra* (8th ed.). New York: Brooks/Cole.
3 Allenby , R. B. J. T. (1991). *Rings, fields and groups* (2nd ed.). New York: Elsevier.
4 Recent research articles.

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MATH -7127 **Theory of Group Graphs** **3 (3+0)**

Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among other fields, to solve problems that can be modelled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including group theory. The purpose of this course is two-fold: we will formally study fundamental concepts in graph theory such as flows and connectivity, the concept of matrices in graphs like Incidence matrix, Adjacency matrix, Cycle matrix and then we will examine some interactions between graphs and groups. Some problems concerning groups are best attacked by using graphs. Although in this context a graph is hardly more than a visual or computational aid, its use does make the presentation clearer and the problems more manageable. The methods are useful both in theory and in practice: they help us to prove general results about groups and results about individual groups. The course aims at familiarizing the students with graphs of group actions, graphical representations of mobius, orthogonal Affine and Euclidean groups.

Contents

1. Graphs
2. Graphs of group actions
3. Projective special linear group
4. Its action on real
5. Rational fields
6. Irrational fields
7. Graphical representations of mobius
8. Orthogonal Affine
9. Euclidean groups

Recommended Texts

1. Magnus, W., Karrass, A., Solitar, D. (2004). *Combinatorial group theory: Presentations of groups in terms of generators and relations* (Revised ed.). New York: Dover Publications.
2. Bollobás B. (1979). *Graphs and groups. In: graph theory. Graduate texts in mathematics*, vol 63. New York: Springer

Suggested Readings

1. Coxeter, H. S. M., Moser, W. O. J. (1980). *Generators and relations for discrete groups* (4th ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Bondy, J. A., Murty, U. S. R. (2008). *Graph theory, Graduate texts in mathematics* (1st ed.). New York: Springer-Verlag
3. Recent research articles.

MATH -7128 **Advanced Complex Analysis-I** **3(3+0)**

This course is intended both for mathematics students continuing to honours work and for other students using mathematics at a high level in theoretical physics, engineering and information technology, and mathematical economics. This course introduces the calculus of complex functions of a complex variable. It turns out that complex differentiability is a very strong condition and differentiable functions behave very well. Integration is along paths in the complex plane. The central result of this spectacularly beautiful part of mathematics is Cauchy's Theorem guaranteeing that certain integrals along closed paths are zero. This striking result leads to useful techniques for evaluating real integrals based on the 'calculus of residues. The course presents an introduction to some topics of contemporary complex analysis, in particular spaces of analytic functions, quasiconformal mappings, univalent functions. The purpose is to prepare the student to independent work in these topics and especially to use the methods of complex analysis in other areas of mathematics, (for example harmonic analysis and differential equations) as well as in applied areas (fluid dynamic, signal analysis, statistics). The content may vary, dependent on the needs and interests of the students.

Contents

1 Analytic continuation
2 Equation of continuity
3 Uniform boundedness
4 Normal and compact families of analytic functions
5 External problems
6 Harmonic functions and their properties
7 Green's and von Neumann functions and their applications
8 Harmonic measure conformal mapping
9 The Riemann mapping theorem
10 The Kernel function
11 Functions of several complex variables

Recommended Texts

1 Hille, E. (2006). *Analytic function theory*, American Mathematical Society.
2 Roland, S. P., & Laura, A. A. (2003). *Conformal mapping: Methods and applications*, New York: Dover Publications.

Suggested Readings

1 Nehari, Z. (1982). *Conformal mappings*, New York: Dover Publications.
2 Sansone, G. & Gerretsen, J. (1969). *Lectures on the theory of functions of a complex variable*, Vol. 2, Netherlands: Wolters-Noordhoff Publishing.
3 Roland, S. P., & Laura, A. A. (2003). *Conformal mapping: Methods and applications*, New York: Dover Publications.
4 Recent research articles.

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MATH -7129 **Advanced Complex Analysis-II** **3(3+0)**

This course is design to understand the modulus of a Complex valued function and results regarding that. To Understand and develop manipulation skills in the use of Rouche's theorem. To Understand certain theorems like Inverse Function theorem, Hardmards three circle theorem. To understand and learn to use Argument Principle. To understand the principal of Analytic Continuation and the concerned results. To study the functions with positive real part. To understand Gamma and Zeta functions, their properties and relationships. To understand the Harmonic functions on a disc and concerned results. To understand the factorization of entire functions having infinite zeros. To understand range of analytic functions and concerned results. To understand univalent functions.

Contents

1 Holomorphic functions: Review of 1-variable theory
2 Real and complex differentiability
3 Power series
4 Complex differentiable Functions
5 Cauchy integral formula for a polydise
6 Chauchy inequalities
7 The maximum principle
8 Extension of analytic functions: Hartogs figures
9 Hartogs theorem, Domains of holomorphy
10 Holomorphic convexity, theorem of Cartan Thullen
11 Levi-convexity: The Levi form, Geometric interpretation of its signature
12 E.E. Levi's theorem, Connections with Kahlerian geometry
13 Elementary properties of plurisubharmonic functions
14 Introduction to Cohomology: Definition and examples of complex manifolds.
15 The d.operators,
16 The Poincare Lemma and the Dolbeaut Lemma,
17 The Cousin problems, introduction to Sheaf theory.

Pre-requisition: Advanced Complex Analysis-I

Recommended Texts

1 Kunihiko, K. (2007). *Complex Analysis, Cambridge studies in advanced mathematics*, Cambridge: Cambridge University Press.
2 Klaus, F., Hans, G. G. (2010). *From holomorphic functions to complex manifolds*, New York: Springer.

Suggested Readings

1 Field, M. (1982). *Several complex variables and complex manifolds*, Cambridge: Cambridge University Press.
2 Grauert, H. and Fritsche, K. (1976). *Several complex variables*, New York: Springer Verlag.
3 Recent research articles.

MATH-7130 **Variational Inequalities** **3 (3+0)**

Variational inequality theory is a powerful unifying methodology for the study of equilibrium problems. 1 Variational inequality theory was introduced by Hartman and Stampacchia (1966) as a tool for the study of partial differential equations with applications principally drawn from mechanics. Such variational inequalities were infinite-dimensional rather than finite-dimensional as we will be studying here. The breakthrough in finite-dimensional theory occurred in 1980 when Dafermos recognized that the traffic network equilibrium conditions as stated by Smith (1979) had a structure of a variational inequality. The variational method is a powerful tool to investigate states and processes in technical devices, nature, living organisms, systems, and economics. The power of the variational method consists in the fact that many of its statements are physical or natural laws themselves. The essence of the variational approach for the solution of problems relating to the determination of the real state of systems or processes consists in the comparison of close states. The selection criteria for the actual states must be such that all the equations and conditions of the mathematical model are satisfied. The first variational theory was the Lagrange theory created to investigate the equilibrium of finite-dimensional mechanical systems under holonomic bilateral constraints (bonds). The selection criterion proposed by Lagrange is the admissible displacement principle. In accordance with this principle, the work of the prescribed forces on infinitesimally small, kinematically admissible displacements is zero. It is known that equating the virtual work performed for potential systems to zero is equivalent to the stationarity conditions for the total energy of the system. The transition from bilateral constraints to unilateral ones was performed by O. L. Fourier. Fourier demonstrated that the virtual work on small disturbances of a stable equilibrium state of a mechanical system under unilateral constraints must be positive (or, at least, nonnegative). Therefore, for such a system the corresponding mathematical model is reduced to an inequality and the problem becomes nonlinear.

Contents

1 Variational problems
2 Existence results for the general implicit variational problems
3 Implicit Ky Fan's inequality for monotone functions
4 Hartman-Stampacchia theorem for monotone for compact operators
5 Selection of fixed points by monotone functions
6 Variational and quasi variational inequalities for monotone operators

Recommended Texts

1 Goh, C. J. (2000). *Duality in optimization and variational inequalities (Optimization theory & applications)* (1st Ed.). Florida: Taylor & Francis.
2 Baiocchi, C. & Capelo, A. (1984). *Variational and quasi-variational inequalities*. New York: Wiley.

Suggested Readings

1 Mosco, V. (1976). *Implicit Variational problems and quasi variational inequalities*, *Lecture Notes in Mathematics*-543. New York: Springer-Verlage, Berlin.
2 Kravchuk, A. S. & Neittaanmaki, P. J. (2007). *Variational and quasi-variational inequalities in mechanics*. New York: Springer-Verlage, Berlin.
3 Recent research articles.

MATH -7131 **Field Extensions and Galois Theory** **3(3+0)**

This course is an introduction to the theory of field extensions and Galois theory. The purpose of Galois theory is to study polynomials at a deep level by using symmetries between the roots. This is a pervasive theme in modern mathematics, and Galois theory is traditionally where one first encounters it. Galois theory also enables us to prove (despite regular claims to the contrary) that there is no ruler and compass construction for trisecting an angle. On successful completion of the course the students should be able to, show familiarity with the concepts of ring and field, and their main algebraic properties, correctly use the terminology and underlying concepts of Galois theory in a problem-solving context; reproduce the proofs of its main theorems and apply the key ideas in similar arguments, calculate Galois groups in simple cases and to apply the group-theoretic information to deduce results about fields and polynomials.

Contents

1 Extension of fields: Elementary properties
2 Simple extensions
3 Algebraic extensions
4 Factorization of polynomials
5 Splitting fields
6 Algebraically closed fields
7 Separable extensions
8 Galois Theory: Automorphism of fields
9 Normal extensions
10 The fundamental theorem of Galois theory
11 Norms and traces
12 The primitive element theorem Lagrange's theorem
13 Nominal bases,
14 Applications Finite fields
15 Cyclotomic extension of rational number field
16 Cyclic extensions
17 Wedderburn's Theorem
18 Ruler-and-Compasses Construction, Solution by Radicals

Recommended Texts

1 Fraleigh, J. B., Brand, N. (2020). *A first course in abstract algebra* (8th ed.). London: Pearson.
2 Weintraub, S. (2009). *Galois Theory* (2nd ed.). New York: Springer-Verlag.

Suggested Readings

1 Rotman, J. J. (1995). *A introduction to the theory of groups* (4th ed.). New York: Springer-Verlag.
2 Herstein, I. N. (1975). Topics in Algebra (2nd ed.). New York: John Wiley
3 Recent research articles.

MATH-7132 **Harmonic Analysis** **3 (3+0)**

Harmonic analysis is the art of decomposing functions and operators into simpler building blocks that can be analyzed separately and then reassembled together. The exponential functions provide an excellent analysis tool, Fourier analysis is the study of such decompositions. Fourier (or harmonic) analysis is a discipline which lies in the intersection of classical and functional analysis and has many applications to differential equations, operator theory, probability and statistics, number theory, and many other areas of mathematics, physics, and engineering. It plays a central role in mathematics because its main idea is to extend decomposition of a vector in an orthogonal basis of a finite dimensional space to orthogonal series expansions or analogous integral representations of functions. The course will enhance research, inquiry and analytical thinking abilities. Following topics will be studied.

Contents

1 Topology, Sets and Topologies
2 Separation axioms and related theorems
3 The Stone- Weierstrass theorem
4 Cartesian products and weak topology
5 Banach spaces, Normed linear spaces
6 Bounded linear transformations, Linear functional
7 The weak topology for X*, Hilbert space
8 Involution on ß (H), Integration, The Daniell integral
9 Equivalence and measurability, The real LP –spaces
10 The conjugate space of LP
11 Integration on locally compact Hausdorff spaces
12 The complex LP –spaces, Banach Algebras
13 Definition and examples, Function algebras
14 Maximal ideals, Spectrum, adverse Banach algebras
15 Elementary theory, The maximal ideal space of a commutative Banach algebra
16 Some basic general theorems

Recommended Texts

1 Deitmar, A., & Echterhoff, S. (2014). *Principles of harmonic analysis*. New York: Springer.
2 Katznelson, Y. (2004). *An introduction to harmonic analysis*. Cambridge: Cambridge University Press.

Suggested Readings

1 Varadarajan, V. S. (1999). *An introduction to harmonic analysis on semisimple lie groups*. New York: Cambridge University Press.
2 Heil, C. (2009). *Introduction to harmonic analysis*. Boston: Birkhäuser.
3 Wang, B., Huo, Z., Guo, Z., & Hao, C. (2011). *Harmonic analysis method for nonlinear evolution equations, I*. World Scientific.

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MATH-7133 **Dynamic Inequalities on Time Scales** **3 (3+0)**

The theory of time scales, which has recently received a lot of attention, was introduced by Stefan Hilger in his PhD thesis in 1988 (supervised by Bernd Aulbach) to unify continuous and discrete analysis. Many results concerning differential equations carry over quite easily to corresponding results for difference equations, while other results seem to be completely different in nature from their continuous counterparts. The study of dynamic equations on time scales reveals such discrepancies, and helps avoid proving results twice, once for differential equations and once for difference equations. The general idea is to prove a result for a dynamic equation where the domain of the unknown function is a so-called time scale, which is an arbitrary nonempty closed subset of the reals. By choosing the time scale to be the set of real numbers, the general result yields a result concerning an ordinary differential equation as studied in a first course in differential equations. On the other hand, by choosing the time scale to be the set of integers, the same general result yields a result for difference equations. However, since there are many other time scales than just the set of real numbers or the set of integers, one has a much more general result. This course consists of 6 sections. In section 1 we present preliminaries and basic concepts of time scale calculus and in section 2 we discuss and prove dynamic inequalities on time scales such as Young's inequality, Jensen's inequality, Holder's inequality, Minkowski's inequality, Steffensen's inequality, Hermite-Hadamard inequality and Čebyšv's inequality. Opial type inequalities on time scales and their extensions with weighted functions will be discussed in section 3. In section 4 we present some inequalities of Lyapunov type for some dynamic equations and in section 5 we employ the shift operators δ_{±} to construct delay dynamic inequalities on time scales and use them to derive Halanay type inequalities for dynamic equations on time scales. Using Halanay's inequalities and the properties of exponential function on time scales, we establish new conditions that lead to stability for nonlinear dynamic equations. Finally, in section 6 we discuss Wirtinger type inequalities on time scales and their extensions.

Contents

1. Time scale calculus
2. Basic definitions, differentiation
3. Examples and applications
4. Integration, chain rule, polynomials
5. Further basic results. Dynamic inequalities
6. Gronwall inequality, Holder and Minkowski's inequalities
7. Jensen's inequality

Recommended Texts

1 Bohner M. & Peterson A. (2001). *Dynamic equations on time scales,* Basel: Birkhauser boston, mass.
2 Bohner M. & Peterson A. (2003). *Advances in dynamic equations on time scales.* Basel: Birkhauser boston, mass.

Suggested Readings

1 Lakshmikatham V. Bhaskar T. G. & Devi J.V. (2006). *Theory of Set Differential equations in Metric Spaces,* Cambridge: Cambridge Scientic.
2 Related Research Papers.

MATH-7134 **Set-Valued Analysis** **3 (3+0)**

This course is devoted to a compressed and self-contained exposition of an important part of contemporary mathematics: set-valued analysis. This course covers the fundamentals of mathematical analysis: convergence of sequences and series, continuity, semicontinuity, derivative, integral, sequences and series of functions, uniformity, and the interchange of limit operations for a set-valued mapping. The course aims at familiarizing the students with measurable set-valued functions, continuous, Lipschitz and some special types of selections, fixed point and coincidence theorems, covering set-valued maps, topological degree theory and differential inclusions, Aumann integral and Hukuhara derivative.

Contents

1. Preliminaries
2. Hausdorff-Pompeiu metric
3. Upper and lower semicontinuous
4. Multifunctions
5. Hausdorff-Pompeiu continuity
6. Closed multifunctions, continuous
7. Selections
8. Measurable multifunctions
9. Aumann integral
10. Hukuhara derivative

Recommended Texts

1. Tarafdar, E. U., Chowdhury, M. S. R. (2008). *Topological methods for set-valued nonlinear analysis* (1st ed.). Singapore: World Scientific Publishing.
2. Lakshmikatham, V., Bhaskar, T. G., Devi J. V. (2006). *Theory of set differential equations in metric* Spaces (1st ed.). Cambridge: Cambridge Scientic.

Suggested Readings

1. Chen, G., Huang, X., Yang, X. (2005). *Vector optimization: Set-valued and variational analysis* (1st ed.). New York: Springer-Verlag Berlin Heidelberg.
2. Aubin, J. P., Frankowska, H. (1990). *Set-Valued Analysis* (1st ed.). Basel: Birkhauser Boston.
3. Recent research papers.

MATH-7135 **Fuzzy Algebra** **3(3+0)**

The primary purpose of this course is to introduce students to the important areas of fuzzy set theory and fuzzy logic. No previous knowledge is needed regarding fuzzy set theory or fuzzy logic. But familiarity with classical set theory, and two-valued logic will be helpful. In most real-life applications of any decision making one needs to face many types on uncertainty. While as humans we can deal with this uncertainty with our reasoning prowess it is not clear how to deal with this uncertainty in a system. Fuzzy sets and fuzzy logic gives us one way of representing this uncertainty and reasoning with them. This course is aimed at providing a strong background for the subject. The decomposition theorems of fuzzy sets and the extension principle will be introduced, as well as the use of nonlinear integrals as aggregation tools to deal with fuzzy data. As an indispensable tool in fuzzy decision making, ranking and ordering fuzzy quantities will be discussed.

Contents

1 Introduction, the Concept of Fuzziness, Examples
2 Mathematical Modeling
3 Operations of fuzzy sets
4 Fuzziness as uncertainty
5 Algebra of Fuzzy Sets
6 Boolean Algebra and lattices
7 Equivalence relations and partions
8 Composing mappings
9 Alpha-cuts, Images of alpha-level sets
10 Operations on fuzzy sets
11 Fuzzy Relations, definition and examples
12 Binary Fuzzy relations Operations on Fuzzy relations, fuzzy partitions
13 Fuzzy Semigroups
14 Fuzzy ideals of semigroups, Fuzzy quasi-ideals, Fuzzy bi-ideals of Semigroups
15 Characterization of different classes of semigroups by the properties of their fuzzy ideals fuzzy quasi-ideals and fuzzy bi-ideals
16 Fuzzy Rings, Fuzzy ideals of rings, Prime
17 semiprime fuzzy ideals
18 Characterization of rings using the properties of fuzzy ideals

Recommended Texts

1 Hung, T. N. (2005). *A first course in fuzzy logic* (3rd ed.). Boca Raton: Chapman and Hall/CRC.
2 Ganesh, M. (2006). *Introduction to fuzzy sets and fuzzy* logic (1st ed.). New Jersey: Prentice-Hall.

Suggested Readings

1 Mordeson, J. N., Malik, D. S. (1998). *Fuzzy commutative algebra* (1st ed.). New York: Singapore: World Scientific.
2 Mordeson, J. N., Malik, D. S., Kuroki, N. (2003). *Fuzzy semigroups* (1st ed.). New York: Springer-Verlage Heidelberg.
3 Recent research papers.

MATH-7136 **Advanced Fuzzy Set Theory** **3(3+0)**

Fuzzy Set Theory is a generalization of Classical set Theory. Fuzzy Set Theory extends classical set theory by allowing degrees of membership, capturing uncertainty in real-world data. Fuzzy Set Theory introduced by Lotfi A. Zadeh in the 1960s, it provides a mathematical framework for dealing with imprecise information and vague concepts. The objectives of fuzzy set theory are to model and handle uncertainty and vagueness in data and concepts, providing a mathematical framework to represent imprecise information accurately.

Contents

1 Introduction and history of Fuzzy Sets
2 Operations of Fuzzy Sets
3 L-R membership function
4 Distances
5 Fuzziness of sets
6 T-norm and T-conorm
7 Archimedian T-norm and T-conorm
8 T-norm and T-conorm of algebraic, Einstein, Frank & Hamacher operator
9 Fuzzy Algebraic Aggregation Operator, Fuzzy Einstein Aggregation Operator
10 Entropy
11 Intuitionistic Fuzzy Set
12 Operations of Intuitionistic Fuzzy Set
13 Intuitionistic Fuzzy Algebraic Aggregation Operator, Intuitionistic Fuzzy Einstein Aggregation Operator
14 Azcel-Alsina Operator & sine trigonometric Operator
15 Decision making
16 TOPSIS method for Fuzzy set
17 TOPSIS method for Intuitionistic Fuzzy Set
18 EDAS method

Recommended Texts

1. De Oliveira, J. V., & Pedrycz, W. (Eds.). (2007). *Advances in fuzzy clustering and its applications.* John Wiley & Sons.
2. Kiler, G. J. and Yuan , B.. (1995), *Fuzzy Logic and Systems: Theory and Applications*, Prentice Hall
3. Pedrycz, W., Ekel, P., & Parreiras, R. (2011). *Fuzzy multicriteria decision-making: models, methods and applications*. John Wiley & Sons.

Suggested Readings

1. Sivanandam, S. N., Sumathi, S., & Deepa, S. N. (2007). *Introduction to fuzzy logic using MATLAB*, Springer-Verlag Berlin Heidelberg.
2. Szmidt, E. (2014). *Distances and similarities in intuitionistic fuzzy sets* (Vol. 307). Switzerland: Springer International Publishing.

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MATH -7137 **Magnetohydrodynamics-I** **3(3+0)**

Magnetohydrodynamics (MHD); also magneto-fluid dynamics or hydromagnetics is the study of the magnetic properties and behavior of electrically conducting fluids. Examples of such magneto-fluids include plasmas, liquid metals, salt water, and electrolytes. This requires an acquaintance with several areas of theoretical physics including electromagnetism, MHD pumps, biological physics, ultrasonic fields and fluid mechanics. The purpose of this subject is to explain the fundamental principles of Magnetohydrodynamics (MHD). The basic derivations of model equations representing the MHD fundamental principles are also the main focus of this course as well along with the classifications. Specifically it contains the impact on the particle motions under MHD impact for the momentum generation towards the field equations. Index notations for the MHD are explained in Euclidean Tensors and Gaussian. Stability of the MHD terms in the field equations will also be considered in the scaling analysis or basic parameters to represent in a non-dimensional quantities. These model equations for MHD will be solved by using appropriate mathematical techniques in mathematics. Further, the main focus will be on MHD boundary value problems specifically used in heat and mass transfer to check the impact of MHD on different boundaries allocated in the particle motions.

Contents

1. Basic Equations: Equations of electrodynamics
2. Equations of fluid dynamics, Ohm's law equations of magnetohydrodynamics
3. Motion of an Incompressible Fluid
4. Motion of a viscous electrically conducting fluid with linear current flow, steady state motion along a Magnetic field, wave motion of an ideal fluid
5. Small amplitude MHD Waves: Magneto-sonic waves, Alfve's waves
6. Damping and excitation of MHD waves
7. Characteristics lines and surfaces
8. Simples Waves and Shock Waves in Magnetohydrodynamics
9. Kinds of simple waves, distortion of the profile of a simple wave, discontinuities, simple ad shock waves in Relativistic magnetohydrodynamics
10. Stability and strucure of shock waves
11. Discontinuities in various quantities
12. Piston problem, oblique shock waves

Recommended Texts

1. Hans, J. P. & Stefaan, P. (2004). *Principles of magnetohydrodynamics: With applications to laboratory and astrophysical plasmas*. Cambridge: Cambridge University Press
2. Davidson, P. A. (2016). *Introduction to magnetohydrodynamics* (2nd ed.). Cambridge: Cambridge University Press

Suggested Readings

1. George, W. & Arthur, S. (2006). *Engineering magnetohydrodynamics*. New York: Mac-Graw Hills
2. Khiezer, A. I. (1975). *Plasma Electrodynamics*. Oxford: Pergamon Press
3. Anderson, J. E. (1975). *Magnetohydrodynamics shock waves*. Cambridge: M.I.T Press
4. Recent research articles.

MATH -7138 **Magetohydrodynamics-II** **3 (3+0)**

The aim of this course is to give students a solid grounding in fundamental plasma physics. Magnetic fields are routinely used in industry to heat, pump and levitate liquid metals. There is the terrestrial magnetic field that is maintained by fluid motion in the earth's core, the solar magnetic field, which generates sunspots and solar flares, and the galactic field that influences the formation of stars. This introductory text on magnetohydrodynamics (MHD) (the study of the interaction of magnetic fields and conducting fluids) is intended to serve as an introductory text for advanced undergraduates and graduate students in physics, applied mathematics and engineering. The material in the text is heavily weighted toward incompressible flows and to terrestrial (as distinct from astrophysical) applications. The final sections of the text, which outline the latest advances in the metallurgical applications of MHD, make the book of interest to professional researchers in applied mathematics, engineering and metallurgy. The word "magnetohydrodynamics" is derived from magneto- meaning magnetic field, hydro- meaning water, and dynamics meaning movement. The field of MHD was initiated by Hannes Alfvén for which he received the Nobel Prize in Physics in 1970. The fundamental concept behind MHD is that magnetic fields can induce currents in a moving conductive fluid, which in turn polarizes the fluid and reciprocally changes the magnetic field itself. The set of equations that describe MHD are a combination of the Navier–Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations must be solved simultaneously, either analytically or numerically.

Contents

1 Flow of Conducting Fluid past Magnetized Bodies
2 Flow of an ideal fluid past magnetized bodies
3 Fluid of finite electrical conductivity flow past a magnetized body
4 Dynamo Theories: Elsasser's theory
5 Bullard's theory, Earth's field Turbulent motion and dissipation
6 Vorticity analogy, Ionized Gases, Effects of molecular structure
7 Currents in a fully ionized gas, partially ionized gases
8 Interstellar fields, dissipation in hot and cool clouds

Pre-requisition: Magetohydrodynamics-I

Recommended Texts

1 Hans, J. P. G. & Stefaan, P. (2004). *Principles of magnetohydrodynamics: With applications to laboratory and astrophysical plasmas*. Cambridge: Cambridge University Press
2 Davidson, P. A. (2016). *An introduction to magnetohydrodynamics* (2nd ed.). Cambridge: Cambridge University Press

Suggested Readings

1 George, W. & Arthur, S. (2006). *Engineering magnetohydrodynamics*. New York: Mac-Graw Hills
2 Khiezer, A. I. (1975). *Plasma Electrodynamics*. Oxford: Pergamon Press
3 Anderson, J. E. (1975). *Magnetohydrodynamics shock waves*. Cambridge: M.I.T Press
4 Recent research articles.

MATH -7139 **Advanced Analytical Dynamics-I** **3(3+0)**

Dynamics is traditionally defined as the classical study of motion with respect to the physical causes of motion, that is, forces and moments. Kinematics, on the other hand, is concerned with the study of motion without respect to the underlying physical causes. In this sense, kinematics is really a fundamental prerequisite upon which dynamics is constructed. For the purposes of this text, the terms dynamics and mechanics are taken to be synonymous. The choice of which term is used is based more on the academic community than on a strict technical distinction. The engineering community typically adopts the term dynamics and the physics and applied mathematics communities typically adopt the term mechanics. Analytical dynamics, or more briefly dynamics, is concerned about the relationship between motion of bodies and its causes, namely the forces acting on the bodies and the properties of the bodies, particularly mass and moment of inertia. The term dynamics is predominantly used in this text.

Contents

1 Equations of dynamic and its various forms
2 Equations of Langrange
3 Equations of Euler
4 Jacobi's elliptic functions and the qualitative and quantitative solutions of the problem of Euler and Poisson
5 The problems of Langrange and Poisson
6 Dynamical system
7 Equations of Hamilton and Appell
8 Hamilton-Jacobi theorem
9 Separable systems
10 Holder's variational principle and its consequences

Recommended Texts

1 Edmund, T. W. (2010). *A treatise on the analytical dynamics of particles and rigid bodies: With an introduction to the problem of three bodies*. London: FQ Books.
2 Bhat, R. B., and Antonio Lopez-Gomez. (2001). *Advanced dynamics* (1st Ed.). New Dehli: Narosa.

Suggested Readings

1 Pars, L. A. (1981). *A treatise on analytical dynamics*. Oxfords: Bow Pub.
2 Recent research articles.

MATH -7140 **Advanced Analytical Dynamics-II** **3(3+0)**

Scope of the course is to introduce students to the theories and methods of modern earthquake geotechnical engineering. The first part of the course is devoted to illustrate fundamental notions of seismology on the origin of earthquakes and on measurement of their size through the concepts of macroseismic intensity and magnitude. Basic notions of seismometery will then be introduced together with the definition of ground motion parameters including the concept of response spectrum. The course will then proceed with the study of seismic hazard at a single site or at an extended territory and on the definition of the design earthquake using both the probabilistic and the deterministic approach. The last part of the course is dedicated to the illustration of basic notions of elastodynamics and seismic wave propagation in a continuum. The main is to Provide a unique bridge between the foundations of analytical mechanics and application to multi-body dynamical systems. To study established principles in mechanics are presented in a thorough and modern way. The course build up from general mathematical foundations, an extensive treatment of kinematics, and then to a rigorous treatment of conservation and variational principles in mechanics. Parallels will be drawn between the different approaches, providing the reader with insights that unify his or her understanding of analytical dynamics. Additionally, a unique treatment will be discussed on task space dynamical formulations that map traditional configuration space representations into more intuitive geometric spaces.

Contents

1 Groups of continuous transformation and Poincare's equations
2 Systems with one degree of freedom
3 Singular points
4 Cyclic characteristics of systems with a degree of freedom
5 Ergodie theorem
6 Metric indecompossability stability of motion

Recommended Texts

1 Edmund, T. W. (2010). *A treatise on the analytical dynamics of particles and rigid bodies: With an introduction to the problem of three bodies*. London: FQ Books.
2 Bhat, R. B., and Antonio Lopez-Gomez. (2001). *Advanced dynamics* (1st Ed.). New Dehli: Narosa.

Suggested Readings

1 Pars, L. A. (1981). *A treatise on analytical dynamics*. Oxfords: Bow Pub.
2 Recent research articles.

MATH-7141	General Relativity	3 (3+0)

The students shall master calculating with tensors and differential forms. They shall also be able to describe physical phenomena in different coordinate systems and to transform from one coordinate system to another. They shall be familiar with covariant derivative and covariant Lagrangian dynamics, geodesic curves, and be able to calculate the components of the Riemann curvature tensor from a given line element. They shall also be able to solve Einstein's field equations for static spherically symmetric problems and for isotropic and homogeneous cosmological models. They shall master calculating the relativistic frequency shifts for sources moving in a gravitational field, as well as the bending of light passing a spherical mass distribution. The students shall also be able to give a mathematical description of gravitational waves, as well as cosmological models in the context of general relativity.

Contents

1 Review of special relativity, tensors and field theory
2 The principles on which general relativity is based
3 Einstein's field equations obtained from geodesic deviation
4 Vacunm equation
5 The Schwarzschild exterior solution
6 Solution of the Einstein-Maxwall field equations
7 The Schwarzschild interior solution
8 The Kerr-Newmann solution (without derivation)
9 Foliations relativistic corrections to Newtonian gravity
10 Black holes, the Kruskal and Penrose diagrams
11 The field theoretic derivation of Einstein's equations
12 Weak field approximations and gravitational waves
13 Kaluza-Klein theory, isometrics. conformal transformations
14 Problems of "quantum gravity"

Recommended Texts

1 James, J. C. (2011). *The geometry of spacetime: An introduction to special and general relativity*. New York City: Springer
2 Abhay, A. (2006). *100 Years of Relativity: Space-time structure einstein and beyond*. Singapore: World Scientific Pub Co Inc

Suggested Readings

1 Qadir, A. (1990). *Relativity: An introduction to the special theory*. Singapore: World Scientific
2 Misner, C. W., Thorne, K. S. and Wheeler, J. A. (1974). *Gravitation*. New York: W.H. Freeman
3 Hawking, S. W. and Ellis, G. F. R. (1972). *The large scale structure of spacetime*. Massachusetts: Academic Press
4 Recent research articles.

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MATH -7142 **Elastodynamics-I** **3(3+0)**

Scope of the course is to introduce students to the theories and methods of modern earthquake geotechnical engineering. The first part of the course is devoted to illustrate fundamental notions of seismology on the origin of earthquakes and on measurement of their size through the concepts of macroseismic intensity and magnitude. Basic notions of seismometery will then be introduced together with the definition of ground motion parameters including the concept of response spectrum. The course will then proceed with the study of seismic hazard at a single site or at an extended territory and on the definition of the design earthquake using both the probabilistic and the deterministic approach. The last part of the course is dedicated to the illustration of basic notions of elastodynamics and seismic wave propagation in a continuum. These concepts will be applied to the study of local site response and of some well-known phenomena of seismic geotechnical risk such as co-seismic instability of natural slopes, cyclic mobility and liquefaction. The techniques will be developed through the boundary element research using the direct time stepping approach or the integral transform approach advanced the theory of elastic waves in solids, their time dependent behavior, and applications, especially in the area of soil-structure interaction and earthquake engineering.

Contents

1 Tensor Analysis
2 Cartesian tensors
3 Orthogonal rotation of axes
4 Transformation equations
5 Translation and rotation
6 Different orders of tensors. Algebra of tensors
7 contraction of tensors
8 Inner and outer multiplication of tensors
9 Symmetric and anti-symmetric tensors. Different types of tensors
10 Tensor Calculus. Differentiation and integration of tensors, application to vector analysis
11 Integral theorems in tensor form
12 Deviators, types of solid Material, Stress vector and stress tensor, Analysis of strain, displacement vector
13 Lagrangian strain tensor, Physical interpretation of strain components. Basic equation of theory of Elasticity
14 Generalized Hooke's law. Types of bodies
15 Physical interpretation of Lame's constants , Navier's equation.

Recommended Texts

1 Shah, N. A. (2005). *Vector and tensor analysis*. Lahore: A-One publisher.
2 Zaman, F. D. (1987). *An introduction to elastodynamic*, Islamabad: National Academy of Higher Education

Suggested Readings

1 Graff, K. F. (1991). *Wave motion in elastic solids*. New York: Dover Publication Inc.
2 Recent research articles.

MATH -7143 **Elastodynamics-II** **3(3+0)**

The aim of this course is to give students advance topics in elastodynamics. The development of linear elastodynamics in pure stress-based formulation began over half-a-century ago as an alternative to the classical displacement-based treatment that came into existence two centuries ago in the school of mathematical physics in France. While the latter approach – fundamentally based on the Navier displacement equation of motion – remains the conventional setting for analysis of wave propagation in elastic bodies, the stress-based formulation and the advantages it offers in elastodynamics and its various extensions remain much less known. Since the key mathematical results of that formulation, as well as a series of applications, originated with J. Ignaczak in 1959 and 1963, the key relation is named the Ignaczak equation of elastodynamics. This course presents the main ideas and results in the stress-based formulation from a common perspective, including (i) a history of early attempts to find a pure stress language of electrodynamics, (ii) a proposal to use such a language in solving the natural traction initial-boundary value problems of the theory, and (iii) various applications of the stress language to elastic wave propagation problems. Finally, various extensions of the Ignaczak equation of elastodynamics focused on dynamics of solids with interacting fields of different nature (classical or micropolar thermoelastic, fluid-saturated porous, piezoelectro-elastic) as well as nonlinear problems are included.

Contents

1 Derivation of equation of motion
2 Helmotz theorem
3 Components of displacement in terms of potentials
4 Strain components, stress components
5 Waves and vibrations in strings, Waves in long string
6 Reflection and transmission at boundaries
7 Free vibration of a finite string, Forced vibration of a string,
8 The string on an elastic base dispersion
9 Pulses in a dispersive media
10 The string on a viscous sub grade

Recommended Texts

1 Shah, N. A. (2005). *Vector and tensor analysis*. Lahore: A-One publisher.
2 Zaman, F. D. (1987). *An introduction to elastodynamic*, Islamabad: National Academy of Higher Education

Suggested Readings

1 Graff, K. F. (1991). *Wave motion in elastic solids*. New York: Dover Publication Inc.
2 Recent research articles.

MATH-7144 **Advanced Heat Transfer** **3 (3+0)**

The aim of the course is to provide the fundamental theory for the analysis of heat transfer processes occurring in boilers, condensers, cooling towers & furnaces. This course is intended as a one semester course for post graduate students. Topics to be covered include basic concepts laws in heat transfer, differential and integral formulation of the energy equations, exact solution of one-dimensional flow problems, boundary layer flow, approximate solutions using the integral method, heat transfer in channel flow, correlation equations in forced & free convection for flat plates, flows over a flat plate with constant and variable temperature & heat flux conditions.

Contents

1. Introduction to modes of heat transfer: Convection, Conduction, Radiation.
2. Basic Laws: First and second laws of thermodynamics, Fourier law of heat conduction, Newton's law of cooling
3. The heat diffusion equation, Boundary and initial conditions.
4. One dimensional steady state conduction with and without heat generation: The plane wall, The radial systems
5. Energy equation, Boundary layer approximation for energy equation
6. Laminar velocity boundary layer, Thermal boundary layer, Fluid friction and heat transfer
7. Free convection boundary layer flow over a vertical flat Plate:
8. Similarity solution for an impermeable flat plate with a variable wall temperature and with variable surface heat flux
9. Flat plate with a variable wall temperature in a stratified environment
10. Flat plate with a sinusoidal wall temperature
11. Mixed convection boundary layer flow along a vertical flat Plate: Flat plate with constant wall temperature and constant heat flux.
12. Mixed convection boundary layer flow along a vertical flat plate with constant wall temperature in parabolic coordinates
13. Effect of Prandtl number on mixed convection boundary layer flow along a vertical flat plate with constant wall temperature
14. Conjugate heat transfer over vertical and horizontal flat plates

Recommended Texts

1. Ioan Pop, Derek B. Ingham (2001). *Convective Heat Transfer*. Elsevier Science Ltd
2. Sadik Kakac & Yaman, Yener. (2013). *Convective heat transfer* (3rd Ed.). Florida: CRC Press
3. Frank P. Incropara, David P. De Witt (1985) *Fundamentals of Heat and Mass Transfer.* John Wiley & Sons

Suggested Readings

1. Holman, J. P. (2010). *Heat transfer.* (10th Ed.). New York: McGraw-Hill
2. Kays, W. M., & Crowfard, M. E. (2005). *Convective heat & mass transfer* (4th Ed.). New York: McGraw-Hill.
3. Recent research articles.

Chairman
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MATH -7145 **Cosmology** **3 (3+0)**

The main objectives of this subject are to offer students a general understanding, at an elementary level, of cosmology from the observational and theoretical perspectives and widen student's view on forefront knowledge and enhance their independent learning skills. The students shall master calculating with differential forms. They shall also be able to describe physical phenomena in different coordinate systems and to transform from one coordinate system to another. They shall be familiar with covariant derivative and covariant Lagrangian dynamics, geodesic curves, and would be able to calculate the components of the curvatures in different forms for a given line element. They shall also be able to solve different fields equations for static spherically symmetric and non symmetric problems and for isotropic, non isotropic and homogeneous cosmological models. They shall be able in calculating the relativistic frequency shifts for sources moving in a gravitational field, as well as the bending of light passing different spherical mass distributions. The students shall also be able to give a mathematical description of gravitational waves, as well as cosmological models in the context of general relativity.

Contents

1 Review of Relativity, historical background
2 Astronomy; Astrophysics Cosmology
3 The cosmological principle and its strong form
4 The Einstein and DeSitter universe models
5 Measurement of cosmic distance, The Hubble law and the Friedmann models
6 Steady state models.The hot big bang model
7 The microwave background, Discussion of significance of a start of time
8 Fundamentals of high energy physics
9 The chronology and composition of the universe
10 Non-baryonic dark matter, Problems of the standard model of cosmology
11 Bianchi space-time's, Mixmaster models, Inflationary cosmology
12 Further developments of inflationary models
13 Kaluza-Klein cosmologies, Review of material

Recommended Texts

1 Steven, L. W. (2008). *Cosmology*. Oxford: Oxford University Press
2 Peter, S. (2010). *Extragalactic astronomy and cosmology: An introduction*. New York: Springer

Suggested Readings

1 Peebles, P. J. E. (1993). *Principles of Physical Cosmology*. New Jersey: Princeton University Press
2 Ryan, M. P. Jr., and Shepley, L. C. (1975). *Homogeneous relativistic cosmologies*. New Jersey: Princeton University) Press.
3 Recent research articles.

MATH -7146 **Electrodynamics-I** 3(3+0)

Electrodynamics contains the basic concept of ionized based particles in the classical motions. This may refer to Electrohydrodynamics (EHD), also known as electro-fluid-dynamics (EFD) or electro-kinetics, is the study of the dynamics of electrically charged fluids. It is the study of the motions of ionized particles or molecules and their interactions with electric fields and the surrounding fluid. EHD printing technology, one of the printing technologies based on EHD theory can offer high resolution patterning and ejection of highly viscous ink achieve thick patterns, unlike conventional inkjet printing systems, which are thermal bubble and piezoelectric actuators. This is a core first-year graduate class whose main objectives are, to lay down the foundations of the understanding of the field theory, including development of important math skills in applications of tensor algebra, partial differential equations, vector calculus, etc., essential for physicists of all later specializations. The basic principles of electrohydrodynamics (EHD) will be reviewed, including governing equations and boundary conditions, the applicability of EHD models, and averaged equations in alternating external fields. Dimensionless criteria for key aspects of electric and EHD processes will be given for the stability concepts. Theoretical treatments of basic EHD phenomena, such as transient processes, sound waves in an electric field, EHD instabilities, EHD flows, and EHD heat exchange, are main focus of this course for the better understandings of the EHD principles.

Contents

1. Maxwell's equations
2. Electromagnetic wave equation, boundary conditions
3. Waves in conducting and non-conducting media reflection and polarization
4. Energy density and energy flux
5. Lorentz formula
6. Wave guides and cavity resonators
7. Spherical and cylindrical waves
8. Inhomogeneous wave equation
9. Retarded potentials
10. Lenard-Wiechart potentials
11. Field of uniformly moving point charge
12. Radiation from a group of moving charges
13. Field of oscillating dipole
14. Field of an accelerated point charge

Recommended Texts

1. Hehl, F. W. & Yuri, N. O. (2003). *Foundation of classical electrodynamics*. United States: Birkhauser
2. Fulvio, M. (2001). *Electrodynamics*. Chicago: University of Chicago Press

Suggested Readings

1. John, R., Reitz, F. J. M. & Robert, W. C. (2008). *Foundations of electromagnetic theory* (4th ed.). United States: Addison Wesley

MATH -7147 **Electrodynamics-II** **3 (3+0)**

This course is the second in a series on Electrodynamics beginning with Electrodynamics- I. It is a survey of basic electromagnetic phenomena: electrostatics; magnetostatics; electromagnetic properties of matter; time-dependent electromagnetic fields; Maxwell's equations; electromagnetic waves; emission, absorption, and scattering of radiation; and relativistic electrodynamics and mechanics. Electrodynamics is a branch of physics that deals with the effects arising from the interactions of electric currents with magnets, with other currents, or with themselves. The aim of this course is to give knowledge for students to work in the field of electromagnetism. After establishing the unifying connections between seemingly different phenomena in nature such as electromagnetic induction and optics, the basic properties of wave propagation, diffraction and interference will be understandable to work in electromagnetism. Specified modeled problems can be solved easily using mathematical techniques for the simulating aspects. This course will be helpful for the understandings of multi-pole radiation with emphasis on dipole and radiation from accelerated charges for the frequency spectrum of the synchrotron radiation and the theory of radiation attenuation. Lagrange and Hamilton methods in field theory will also be easier to give specified applications in electrodynamics.

Contents

1 General angular and frequency distributions of radiation from accelerated charges
2 Thomson scattering
3 Cherenkov radiation
4 Fields and radiation of localized oscillating sources
5 Electric dipole fields and radiation
6 Magnetic dipole and electric quadruple fields, multipole fields
7 Multipole expansion o f the electromagnetic fields
8 Angular distributions sources of multipole radiation
9 Spherical wave expansion of a vector plane wave
10 Scattering of electromagnetic wave by a conducting sphere

Pre-requisition: Electrodynamics-I

Recommended Texts

1 Hehl, F. W. & Yuri, N. O. (2003). *Foundation of classical electrodynamics*. United States: Birkhauser
2 Fulvio, M. (2001). *Electrodynamics.* Chicago: University of Chicago Press

Suggested Readings

1 J ohn R., Reitz, F. J. M. & Robert, W. C. (2008). *Foundations of electromagnetic theory* (4th ed.). United States: Addison Wesley

MATH-7148 **Perturbation Methods-I** **3 (3+0)**

The purpose and objective of this course is to learn and exercise the applications of perturbation expansion techniques to the solution of differential equations and approximation of integrals. Approximation expansions are generated in the form of asymptotic series. These may not and often do not converge but, in a truncated form of only two or three terms, provide a useful approximation to the original problem. The techniques, being analytical rather than numerical, provide an alternative to a direct computer solution. Awareness of the perturbation approach is sometimes essential even when a direct numerical approach is adopted. Students, teachers and researchers in applied and engineering mathematics should learn this technique to gaining an understanding of most of the powerful perturbation techniques along with their applications. The basic ideas, however, are also applicable to integral equations, integro differential equations, and even to difference equations. In essence, a perturbation procedure consists of constructing the solution for a problem involving a small parameter B, either in the differential equation or the boundary conditions or both, when the solution for the limiting case B = 0 is known. The main mathematical tool used is asymptotic expansion with respect to a suitable asymptotic sequence of functions of B.

Contents

1 Parameter of perturbations
2 Coordinate perturbations, order symbols and gauge functions
3 Asymptotic series and expansions
4 Asymptotic expansion of integrals, integration by parts
5 Laplace's method and Watson's lemma, method of stationary phase and method of steepest descent
6 Straightforward expansions and sources of non-uniformity
7 The Duffing equation, small Reynolds number
8 The method of strained coordinates
9 The Lindstedt – Poincare' method
10 Renormalization method
11 Variation of parameters and method of averaging, examples
12 Method of Multiple scales with examples
13 Flow past a sphere
14 Small parameter multiplying the highest derivative

Recommended Texts

1 Bush, A. W. (1992). *Perturbation methods for engineers and scientists*. Florida: CRC Press.
2 Ali, H. N. (1980). *Introduction to perturbation techniques*. New York: A Willey Inter science Publication John Willey & Sons.

Suggested Readings

1. Giacoglia, G. E. O. (1972). *Perturbation methods in non-linear system*. New York: Springer.
2. Holmes, M. H. (2013). *Introduction to perturbation methods*. New York: Springer.
3. Recent research articles.

MATH-7149 **Perturbation Methods-II** **3 (3+0)**

The purpose and objective of this course is to learn and exercise the applications of perturbation expansion techniques to the solution of differential equations and approximation of integrals. Approximation expansions are generated in the form of asymptotic series. These may not and often do not converge but, in a truncated form of only two or three terms, provide a useful approximation to the original problem. The techniques, being analytical rather than numerical, provide an alternative to a direct computer solution. Awareness of the perturbation approach is sometimes essential even when a direct numerical approach is adopted. Students, teachers and researchers in applied and engineering mathematics should learn this technique to gaining an understanding of most of the powerful perturbation techniques along with their applications. The perturbation methods, especially in connection with differential equations, in order to illustrate certain general features common to many examples will be exercised. The basic ideas, however, are also applicable to integral equations, integro differential equations, and even to difference equations. In essence, a perturbation procedure consists of constructing the solution for a problem involving a small parameter B, either in the differential equation or the boundary conditions or both, when the solution for the limiting case B = 0 is known. The main mathematical tool used is asymptotic expansion with respect to a suitable asymptotic sequence of functions of B.

Contents

1 Approximate Solution of Linear Differential Equations
2 Approximate Solution of Nonlinear Differential Equations
3 Perturbation Series
4 Regular and Singular Perturbation Theory
5 Perturbation Methods for Linear Eigen value Problems
6 Asymptotic Matching, Boundary Layer Theory
7 Mathematical Structure of Boundary Layers: Inner, Outer, and Intermediate Limits Higher-Order Boundary Layer Theory
8 Distinguished Limits and Boundary Layers of Thickness
9 WKB Theory Exponential Approximation for Dissipative and Dispersive Phenomena,
10 Conditions for Validity of the WKB approximation, matched Asymptotic
11 Approximations: WKB Solution of inhomogeneous Linear equations
12 Matched Asymptotic Approximation: Solution of the One-Turning-Point Problem

Recommended Texts

1 Bush, A. W. (1992). *Perturbation methods for engineers and scientists*. Florida: CRC Press.
2 Ali, H. N. (1980). *Introduction to perturbation techniques*. New York: A Willey Inter science Publication John Willey & Sons.

Suggested Readings

1. Giacoglia, G. E. O. (1972). *Perturbation methods in non-linear system*. New York: Springer.
2. Holmes, M. H. (2013). *Introduction to perturbation methods*. New York: Springer.
3. Recent research articles.

MATH-7150 **Viscous Fluids-I** **3(3+0)**

This is a first graduate level course intended to cover the fundamentals of fluid mechanics from an advanced point of view, with emphasis on the mathematical treatment of viscosity effects in laminar flows of a Newtonian fluid. We begin with the Navier Stokes equations and some of its exact solutions available in simplified configurations. Attention is given to the Stokes-flow regime of very low Reynolds numbers, flows with wall and free-shear boundaries, and the effects of pressure gradients, heat transfer and compressibility. We also provide an introduction to the phenomena of instability and transition to turbulence. Observations on Taylor-vortex flows between cylinders of comparatively small but variable length are reported, revealing properties unexplained by older theories. The observed flows are classified as follows: (i) the primary mode which is uniquely possible at small values of the Reynolds number *R*, and which usually develops smoothly with increasing *R*; (ii) secondary modes which are possible only above a respective critical value of *R*, and which are shown to manifest predicted behavior as this value is approached.. Two novel and surprising examples of (ii) are reported. A predicted hysteresis phenomenon is confirmed, relating to morphogenesis of the primary mode between two-cell and four-cell forms as the length of the annulus is varied.

Contents

1 Some examples of viscous flow phenomena
2 Properties of fluids
3 Boundary conditions
4 Equation of continuity
5 The Navier stokes equations
6 The energy equation
7 Orthogonal coordinate systems
8 Dimensionless parameters
9 Velocity considerations; two dimensional considerations, and the stream functions
10 Coutte flows, Paisvilleflow, unsteady duct flows
11 Similarity solutions, some exact analytic solution from the papers
12 Introduction; laminar boundary layers equations; similarity solutions
13 Two dimensional solutions
14 Thermal boundary layer
15 Some exposure will also be given from the recent literature appearing in the journals

Recommended Texts

1 White F. M. (2005). *Viscous fluid flow* (3rd Ed.) New York: McGraw Hill Inc.
2 Schlichting, H. and Gertsen, K. (1991). *Boundary layer theory.* New York: Springer.

Suggested Readings

1 Davidson, P. A. (2001). *An Introduction to magnetohydrodynamics*, Cambridge: Cambridge University Press.
2 Recent research articles

MATH-7151 **Viscous Fluids-II** **3(3+0)**

This is a first graduate level course intended to cover the fundamentals of fluid mechanics from an advanced point of view, with emphasis on the mathematical treatment of viscosity effects in laminar flows of a Newtonian fluid. We begin with the Navier Stokes equations and some of its exact solutions available in simplified configurations. Attention is given to the Stokes-flow regime of very low Reynolds numbers, flows with wall and free-shear boundaries, and the effects of pressure gradients, heat transfer and compressibility. The observed flows are classified as follows: (i) the primary mode which is uniquely possible at small values of the Reynolds number R, and which usually develops smoothly with increasing R; (ii) secondary modes which are possible only above a respective critical value of R, and which are shown to manifest predicted behavior as this value is approached.. Two novel and surprising examples of (ii) are reported. A predicted hysteresis phenomenon is confirmed, relating to morphogenesis of the primary mode between two-cell and four-cell forms as the length of the annulus is varied. We also provide an introduction to the phenomena of instability and transition to turbulence.

Contents

1 Introduction
2 The concept of small disturbance stability
3 Linearized stability; parametric effects in the linear stability theory; transition to turbulences
4 Boundary layer equation in plane flow
5 General solution and exact solutions of the boundary layer equations
6 Thermal boundary layers without coupling of velocity field to the temperature field
7 Boundary layer equations for the temperature field
8 Forced convection; effect of Pr number
9 Similar solution of the thermal boundary layers
10 Thermal boundary layer with coupling of velocity field to the temperature field
11 Boundary layer with moderate wall heat transfer
12 Natural convection effect of dissipation
13 Indirect natural convection; mixed convection
14 Different kinds of boundary layer control
15 Continuous suction and blowing
16 Massive suction and blowing; similar solutions

Recommended Texts

1 White F. M. (2005). *Viscous fluid flow* (3rd Ed.). New York: McGraw Hill Inc.
2 Schlichting, H. and Gertsen, K. (1991). *Boundary layer theory*. New York: Springer.

Suggested Readings

1 Davidson, P. A. (2001). *An introduction to magneto hydrodynamics*. Cambridge: Cambridge University Press.
2 Recent research articles

MATH-7152 **Environmental Heat Transfer** **3 (3+0)**

Environmental heat transfer is a discipline of thermal engineering that concerns the exchange of thermal energy (heat) between different components of the Earth's environment, including the atmosphere, oceans, and land surfaces. This transfer of heat is precarious to the Earth's climate system and influences weather patterns, climate variability, and the overall energy balance of the planet. The main mechanisms of heat transfer in the environment are conduction, convection, and radiation. Topics to be covered include basic concepts in Fluid Mechanics and heat transfer, differential formulation of the continuity, momentum & energy equations, solution of boundary layer flow, heat transfer in channel flow. After completion of this course students will be eligible to understand the effects of temperature differences on the future climate conditions.

Pre-requisite:
Fluid Mechanics and Heat Transfer

Contents

1 Atmosphere, Hydrosphere and the Solar Spectrum
2 Basic Features of Atmosphere and Hydrosphere
3 Fossil Fuels and their impact on Environment
4 Green House Gases and their effects.
5 Global Warming and Climate Change
6 Seasonal Heating or Cooling.
7 Heat and Thermodynamics of the Atmosphere: The Nature of Heat and Kinetic Theory
8 Thermodynamic Equations for Fluids
9 Thermodynamics of Seawater
10 Effects of Temperature Gradient on Climate Change
11 Adiabatic Changes in the Atmosphere.
12 Differences of Temperature Between Cloud Elements
13 Effects of Temperature on Ice-Crystal
14 Climatological Rainfall Distribution over Globe due to temperature differences
15 Heat Balance of Earth-Atmosphere System: Definitions of Heat Source and Sinks
16 Physical Processes Involved in Heat Balance
17 Heat Sources and Sinks from Energy Balance Equation
18 Effects of temperature on unsaturated air and saturated air
19 Nanoparticles and Buongiorno model for convective Heat Transfer

Recommended Texts

1. Saha, K. (2008). *The Earth's atmosphere: its physics and dynamics* (1st ed.), Springer.
2. Nelson, V. C. (2011). *Introduction to renewable energy* (2nd ed.). CRC press.
3. Wallace, J. M., & Hobbs, P. V. (2006). *Atmospheric science: an introductory survey* (2nd ed.). Elsevier.

Suggested Readings

1. Incropera, F. P., & Dewitt, D. P. (1985). *Fundamentals of heat & mass transfer* (2nd ed.). New York: Wiley.
2. Kakac, S., Yener, Y., & Pramuanjaroenkij, A. (2013). *Convective heat transfer* (3rd ed.). CRC press.
3. Pop, I., & Ingham, D. B. (2001). *Convective heat transfer: mathematical and computational modelling of viscous fluids and porous media* (1st ed.). Elsevier.
4. Granger, R. A. (1985). *Fluid mechanics* (1st ed.). New York: Dover Publications Inc.

MATH -7153 **Graph Theory** **3 (3+0)**

Graph theory is one branch of the wide-ranging field known nowadays as combinatorics. It has applications in many different areas, including parts of computer science, operations research including scheduling, network flows and circuit design. Graph theory has been applied to several areas of physics, chemistry, communication science, biology, electrical engineering, operations research, psychology, linguistics, among others fields, to solve problems that can be modeled as discrete objects called graphs. Graph theory is intimately related to different branches of mathematics including group theory, matrix theory, numerical analysis, probability, topology, and combinatorics. Even though some of the problems in graph theory can be described in an elementary way, many of these problems represent a challenge to many researchers in mathematics. Graph theory plays an important role in operating system in solving job scheduling and resource allocation problems. The concept of graph coloring is applied in job scheduling problems of CPU. A graph is symbolic representation of a network and connectivity. It is concerned that how networks can be encoded.

The purpose of this course is two-fold: we will formally study fundamental concepts in graph theory, such as flows and connectivity (Manger's theorem), planarity (coloring), Euclidean and Hamiltonian graphs, and reliability theory; from an applications point of view students we will be able to measure performance objectives of communications networks modeled as probabilistic networks via network reliability theory, by incorporating previous theoretical knowledge acquired during the course.

Contents

1 Fundamental and basic definitions
2 Paths cycles and trees
3 Hamilton cycles and Euler circuits
4 Planer graphs. Flows
5 Connectivity and Matching Network flows
6 Connectivity and Menger's theorem
7 External problems paths and Complete Subgraphs
8 Hamilton path and cycles
9 Colouring, Vertex colouring
10 Edge coloring
11 Graph on surfaces

Recommended Texts

1 Bondy, J. A., & Murty, U. S. R. (2010). *Graph theory with applications*. London: Macmillan
2 Gross, J. L., & Yellen, J. (2005). *Graph theory and its applications*. Cambridge: CRC press.

Suggested Readings

1 Bollobás, B. (2013). *Modern graph theory* (Vol. 184). Berlin: Springer Science & Business Media.
2 Trudeau, R. J. (2013). *Introduction to graph theory. Massachusetts*: Courier Corporation.
3 Chartrand, G. (2006). *Introduction to graph theory*. New York: McGraw-Hill Education.
4 Golumbic, M. C. (2004). *Algorithmic graph theory and perfect graphs*. Netherlands: Elsevier.

MATH -7154 **Approximation Theory** **3(3+0)**

Approximation theory is the branch of mathematics which studies the process of approximating general functions by simple functions such as polynomials, finite elements or Fourier series. It therefore plays a central role in the analysis of numerical methods, in particular approximation of PDE's. The chapter introduces various function spaces, in particular Hölder, Sobolev and Besov smoothness classes. Two important types of inequalities that are essential to build a more general theory are discussed: direct (or Jackson type) inequalities, and inverse (or Bernstein type) inequalities that take into account the smoothness properties of the V_j spaces. In the setting of finite element spaces, such inequalities are classically derived by using the properties of the affine mapping between each element and a reference domain and basic results of polynomial approximation in this reference domain. The primary objective of this course is to lay the theoretical foundation for the wider field of numerical analysis by looking at some of the classical topics in approximation theory. At the end of the course, the students are expected to understand and master theoretical as well as practical issues that arise in approximation of functions by polynomials, trigonometric polynomials, splines and by rational functions

Contents

1 Best approximation in metric spaces
2 Best approximation in normed spaces
3 Least square approximation
4 Rational approximation
5 Haar condition in function spaces
6 Best approximation in function spaces
7 Interpolation stone
8 Weierstrass theorem for scalar valued functions
9 Weierstrass theorem for vector valued functions
10 Spline approximation

Recommended Texts

1 Achieser, N. I. (2004). *Theory of approximation*, New York: Dover Publications.
2 Cheney, W. E. (2000). *Introduction to Approximation Theory* (2nd ed.). Providence: Amer Mathematical Society.

Suggested Readings

1 Powell, M. D. (1981). *Approximation theory and methods*, Cambridge: Cambridge University Press.
2 Holmes, R. B. (1971). *A course on optimization and best approximation, lecture notes in mathematics no. 257*, New York: Springer-Verlag.
3 Recent research articles.

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MATH-7155 **Introduction to Subdivision Scheme** **3 (3+0)**

Promote subdivision that is functional and enhances the knowledge regarding the construction of freeform curves and surfaces. In recent years, subdivision schemes have become an integral part of computer graphics in view of their extensive variety of applications in the field of visualizations, animation and image processing. Subdivision scheme deals with algorithms for free-form curves, surfaces and volumes. It plays a significant role in integrating computers and industry. Smooth curves and surfaces have pivotal importance in the field of air craft manufacturing, movie animation, computer game character design and general product design.

Contents

1 Bilinear interpolation, The direct de Casteljau Algorithm
2 The tensor product approach and its properties
3 Bernstein polynomial, degree elevation, constructing polynomial patches: Ruled surfaces
4 Coons patches, translational surfaces, tensor product interpolation, bicubic Hermite patches
5 Composite Surfaces
6 tensor product B-spline surfaces, Matrix representation
7 Cubic spline interpolation
8 Rational Bezier and B-spline surfaces, surface of revolution
9 COONS and trimmed surface
10 Lofted Surfaces
11 Bezier Triangles: The de Casteljau algorithm, triangular blossoms
12 Bernstein polynomial, derivatives, subdivision, differentiability
13 Nonparametric patches
14 S-Patches. Surfaces with Arbitrary Topology
15 Recursive subdivision curve
16 Properties of Subdivision scheme
17 Types of Subdivision scheme
18 Analysis of Subdivision
19 Smoothness analysis of Binary, Ternary , Quaternary Subdivision scheme
20 Tensor product of Subdivision scheme
21 Smoothness analysis of Subdivision scheme

Recommended Texts

1 Gerald, F. (2002). *Curves and surfaces for CAGD. A practical guide.* (5th Ed.), Burlington: Morgan Kaufmann Publishers
2 Josef, H. & Dieter, L. (1993). *Fundamentals of computer aided geometric design.* Natick: A. K Peter Ltd

Suggested Readings

1 Gerald, F., Josef, H. & Myung, S. (2002). *Handbook of computer aided geometric design.* Netherlands: Elsevier Science.
2 Armin, I., Ewald, Q., Michael, S. F. (2002). *Tutorials on multiresolution in geometric modeling.* Summer School Lecture Notes, New York: Springer-Berlin.
3 Recent research papers.

MATH-7156	Design Theory	3 (3+0)

Design theory is a branch of combinatorics. Its traditional roots are in the design of experiments, but it has found recent applications in cryptography, coding theory and communication networks.
Over the past several decades, algebra has become increasingly important in combinatorial design theory. The flow of ideas has for the most part been from algebra to design theory. Moreover, despite our successes, fundamental algebraic questions in design theory remain open. It seems that new or more sophisticated ideas and techniques will be needed to make progress on these questions. In the meantime, design theory is a fertile source of problems that are ideal for spurring the development of algorithms in the active field of computational algebra. Combinatorial designs are used to determine which patients receive which treatments in such a way that if a given response is observed, then the structure of the design would indicate the treatment that caused it. Modern applications are also found in a wide gamut of areas including; Finite geometry, tournament scheduling, lotteries, mathematical chemistry, mathematical biology, algorithm design and analysis, networking, group testing and cryptography. This Course includes an introduction to Design Theory including a selection of topics from Latin squares, Steiner triple systems, balanced incomplete block designs, graph decompositions. projective and affine designs. The course should allow students subsequently to read further in these areas, and to apply their knowledge of graph theory and design theory to other appropriate fields.

Contents

1 Basic definitions and properties
2 Related structure, The incidence matrix
3 Graphs, residual structures
4 The Bruck - Ryser-Chowla theorem
5 Singer groups and difference sets
6 Arithmetical relations and Hadamard 2- designs
7 Projective and affine planes
8 Latin squares, nets. Hadamard matrices and Hadamard 20 design
9 Biplanes, strongly regular graphs
10 Cameron's theorem and Hadamard 3-desings
11 Steiner triple systems, The Mathieu groups

Recommended Texts

1 Beth, T., Jungnickel, D., & Lenz, H. (1999). *Design theory: Volume 1*. Cambridge: Cambridge University Press.
2 Wallis, W. D. (2016). *Introduction to combinatorial designs*. Boca Raton: CRC Press.

Suggested Readings

1 Cameron, P. J., Van Lint, J. H., & Cameron, P. J. (1991). *Designs, graphs, codes and their links* (Vol. 3). Cambridge: Cambridge University Press.
2 Cameron, P. J. (1999). *Permutation groups* (Vol. 45). Cambridge: Cambridge University Press
3 Lindner, C. C., & Rodger, C. A. (2017). *Design theory*. Florida: CRC press

MATH-7157 **Acoustics** **3(3+0)**

This course introduces acoustics by using the concept of impedance. The course starts with vibrations and waves, demonstrating how vibration can be envisaged as a kind of wave, mathematically and physically. They are realized by one-dimensional examples, which provide mathematically simplest but clear enough physical insights. Then the part 1 ends with explaining waves on a flat surface of discontinuity, demonstrating how propagation characteristics of waves change in space where there is a distributed impedance mismatch.
Describe the characteristics of spaces designed for effective listening, working, learning, and other functions. Describe environmental acoustics in terms of acoustic enhancement and environmental noise control. Relate the scientific principles of acoustic design to the design and construction of comfortable spaces.

Contents

1 Fundamentals of vibrations, Energy of vibration
2 Damped and free oscillations
3 Transient response of an oscillator vibrations of strings
4 Membranes and plates, forced vibrations
5 Normal modes, Acoustic waves equation and its solution
6 Equation of state, equation of cout, Euler's equations
7 Linearized wave equation, speed of sound in fluid
8 Energy density, acoustic intensity, specific acoustic impedance
9 Spherical waves, transmission
10 Transmission from one fluid to another (Normal incidence) reflection at a surface of solid (normal and oblique incidence)
11 Absorption and attenuation of sound waves in fluids
12 Pipes cavities waves guides, under water acoustics

Recommended Texts

1 Everest, F. A., & Pohlmann, K. C. (2015). *Master handbook of acoustics* (5th ed.). New York: McGraw-Hill
2 Kinsler, L. E., Frey, A. R., Coppens, A. B., & Sanders, J. V. (1999). *American: Fundamentals of acoustics*. New York: Wiley-VCH.

Suggested Readings

1 Morse, P. M., & Ingard, K. U. (1986). *Theoretical acoustics*. New Jersey: Princeton university press.
2 Redfern, F. R., & Munson, R. D. (1982). *Acoustic emission source location: a mathematical analysis* (Vol. 8692). Washington: US Department of the Interior, Bureau of Mines.
3 Vorländer, M. (2007). *Auralization: fundamentals of acoustics, modelling, simulation, algorithms and acoustic virtual reality*. New York: Springer Science & Business Media.

MATH-7158 **Combinatorics** **3(3+0)**

Combinatorics is an area of mathematics primarily concerned with counting, both as a means and an end in obtaining results, and certain properties of finite structures. It is closely related to many other areas of mathematics and has many applications ranging from logic to statistical physics, from evolutionary biology to computer science, etc. Combinatorics is a growing field utilized in data science, computer science, statistics, probability, engineering, physics, business management, and everyday life. This course is a great introduction with some specialized topics. In this course, students will become familiar with fundamental combinatorial structures that naturally appear in various other fields of mathematics and computer science. They will learn how to use these structures to represent mathematical and applied questions, and they will become comfortable with the combinatorial tools commonly used to analyze such structures. Given a hypothetical combinatorial object that must satisfy certain properties, students will learn how to prove the existence or non-existence of the object, compute the number of such objects, and understand their underlying structure.

Contents

1. Elementary concepts of several combinatorial structures
2. Recurrence relations and generating functions
3. Principle of inclusion and exclusion
4. Latin squares and SDRs. Steiner system
5. A direct construction, A recursive construction
6. Packing and covering
7. Linear algebra over finite fields
8. Gaussian coefficients, The pigeonhole Principle
9. Some special cases, Ramsey's theorem
10. Bounds for Ramsey numbers and applications
11. Automorphism groups and permutation groups
12. Enumeration under group action

Recommended Texts

1. Lothaire, M., & Lothaire, M. (2002). *Algebraic combinatorics on words* (Vol. 90). Cambridge: Cambridge university press.
2. Stanley, R. P. (2007). *Combinatorics and commutative algebra* (Vol. 41). Cambridge: Springer Science & Business Media.

Suggested Readings

1. Brualdi, R. A. (1977). *Introductory combinatorics*. London: Pearson Education.
2. Brualdi, R. A., & Cvetkovic, D. (2008). *A combinatorial approach to matrix theory and its applications*. Florida: CRC press.
3. Flajolet, P., Lam, T. Y., & Lutwak, E. (1987). *Combinatorial geometries* (Vol. 29). Cambridge: Cambridge University Press.

MATH-7159 **Theory of Majorization** **3(3+0)**

In mathematics, majorization is a preorder on vectors of real numbers. For a vector, we denote by the vector with the same components, but sorted in descending order. Given, we say that weakly majorizes (or dominates) from below written as iff. Many problems arising in signal processing and communications involve comparing vector-valued strategies or solving optimization problems with vector- or matrix-valued variables. Majorization theory is a key tool that allows us to solve or simplify these problems. This course play a fundamental role in nearly all branches of mathematics, inequalities are usually obtained by ad hoc methods rather than as consequences of some underlying "theory of inequalities." For certain kinds of inequalities, the notion of majorization leads to such a theory that is sometimes extremely useful and powerful for deriving inequalities. Moreover, the derivation of an inequality by methods of majorization is often very helpful both for providing a deeper understanding and for suggesting natural generalizations.

Contents

1 Motivation and Basic Definitions
2 Majorization as a Partial Ordering
3 Order-Preserving Functions
4 Partial Orderings Induced by Convex Cones
5 Partial Orderings Generated by Groups of Transformations
6 Majorization for Vectors of Unequal Length
7 Majorization for Infinite Sequences
8 Majorization for Matrices, Lorenz Ordering
9 Majorization and Dilations, Complex Majorization

Recommended Texts

1 Marshall, A. W., Olkin, I., & Arnold, B. C. (1979). *Inequalities: theory of majorization and its applications* . New York: Academic press.
2 Bhatia, R. (2013). *Matrix analysis* (Vol. 169). New York: Springer Science & Business Media.

Suggested Readings

1 Peajcariaac, J. E., & Tong, Y. L. (1992). *Convex functions, partial orderings, and statistical applications.* New York: Academic Press
2 Arnold, B. C. (2012). *Majorization and the Lorenz order: A brief introduction.* New York: Springer Science & Business Media.
3 Jorswieck, E., & Boche, H. (2007). *Majorization and matrix-monotone functions in wireless communications* (Vol. 3). Netherland: Now Publishers Inc.
4 Latest Research Papers.

MATH-7160 **Nilpotent and Solvable Groups** **3 (3+0)**

Abelian groups are some of the easiest to understand and most frequently met groups. They have as natural generalizations nilpotent, polycyclic and solvable groups, which generalizations are used in many different areas of algebra, geometry and topology. The course begins with the series of groups and the classification of (infinite) finitely generated Abelian groups. It continues with some of the main properties and results on nilpotent and solvable groups, illustrated with many examples, especially of linear groups. After introducing the growth function for infinite finitely generated groups and proving that nilpotent groups have polynomial growth, the course ends with the Milnor-Wolf Theorem. The latter theorem formulates a striking dichotomy in the class of solvable groups: such groups are either virtually nilpotent or of exponential growth, Students will learn fundamentals about the various subclasses of solvable groups, about the algebraic and geometric features that they have in common, as well as about those that help to differentiate them.

Contents

1 Normal and subnormal series
2 Abelian and central series, direct products
3 finitely generated Abelian groups, splitting theorems
4 Solvable groups
5 Nilpotent groups
6 Commutators subgroup, derived series, the lower and upper central series
7 Characterzation of finite Nilpotent groups
8 Fitting subgroup
9 Frattini subgroup
10 Dedekind groups
11 Supersolvable groups, solvable groups with minimal condition
12 Subnormal subgroups
13 Minimal condition on subnormal subgroups
14 The subnormal socle, the Wielandt subgroup and Wielandt series
15 T-groups,
16 Power automorphisms
17 Structure and Construction of finite soluble T-Groups

Recommended Texts

1 Kurzweil, H., Stellmacher, B. (2004). *The theory of finite groups* (1st ed.). New York: Springer-Verlag.
2 Roman, S. (2011). *Fundamentals of group theory: An advanced approach* (1st ed.). New York: Springer.

Suggested Readings

1 Robinson, D. (1996). *A course in the theory of groups* (2nd ed.). New York: Springer-Verlag.
2 Doerk, K., Hawkes, T. (1992). *Finite soluble* (1st ed.). Berlin: De Gruyter.
3 Recent research papers.

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MATH-7161 **Mathematical Modeling-I** **3(3+0)**

The main objective of the course is to introduce mathematical modeling, that is, the construction and analysis of mathematical models inspired by real life problems. The course will present several modeling techniques and the means to analyze the resulting systems. Students can expect to acquire the following knowledge and skills at the end of the Masters course: (1) Theory of partial derivative equations, numerical discretization, error analysis; (2) Continuous and discrete optimization, calculus of variations, game theory; (3) Finite or infinite dimensional control theory, optimal control theory, inverse problems; (4) Analysis, simulation and modeling tools that are used in life sciences; (5) Scientific computing, scientific computation, parallel computation, computer-aided design. Students will also acquire knowledge in various applied fields: computer science, biology, physics, mechanics, and economics.

Contents

1 Introduction to Modeling
2 Collection and interpretation of data
3 Setting up and developing models
4 Checking models. Consistency of models
5 Dimensional analysis
6 Discrete models
7 Multivariable models
8 Matrix models
9 Continuous models
10 Modeling rates of changes
11 Limiting models
12 Graphs of functions as models
13 Periodic models
14 Modeling with difference equations
15 Linear
16 Quadratic and Non-Linear Models.

Recommended Texts

1 Edwards, D. and Hamson, M. (1996). *Mathematical modeling skills*. New York: Macmillan Press Ltd.
2 Giordano, F. R., Weir, M. D. and Fox, W. P. (2003). *A first course in mathematical modeling*. New York: Brooks Cole.

Suggested Readings

1 Thomas, W., Mark, B. (2015). *Methods of mathematical modelling: Continuous systems and differential equations*. New York: Springer.
2 Sandip, B. (2014). *Mathematical modeling: Models, analysis and applications*. Florida: CRC Press.
3 Clive L. D. (2004). *Principles of mathematical modeling* (2nd ed.). Cambridge: Academic Press.
4 Recent research articles.

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MATH-7162 **Mathematical Modeling-II** **3(3+0)**

The main objective of the course is to introduce mathematical modeling, that is, the construction and analysis of mathematical models inspired by real life problems. The course will present several modeling techniques and the means to analyze the resulting systems. Students can expect to acquire the following knowledge and skills at the end of the Masters course: (1) Theory of partial derivative equations, numerical discretization, error analysis; (2) Continuous and discrete optimization, calculus of variations, game theory; (3) Finite or infinite dimensional control theory, optimal control theory, inverse problems; (4) Analysis, simulation and modeling tools that are used in life sciences; (5) Scientific computing, scientific computation, parallel computation, computer-aided design. Students will also acquire knowledge in various applied fields: computer science, biology, physics, mechanics, and economics.

Contents

1. Modeling with Differential Equations
2. Exponential growth and decay
3. Linear
4. Non-linear systems of differential equations
5. Modeling with integration
6. Modeling with random numbers
7. Simulating qualitative random variables
8. Simulating discrete random variables
9. Standard models
10. Monte Carlo simulation
11. Fitting models to data
12. Bilinear interpolation and Coons patch

Pre-requisition: Mathematical modeling-I

Recommended Texts

1. Giordano, F. R., Weir, M. D. and Fox, W. P. (2003). *A first course in mathematical modeling*. New York: Brooks/Cole.
2. Dilwyn, E., Mike, H. (2001). *Guide to mathematical modeling (mathematical guides)*. London: Palgrave Macmillan

Suggested Readings

1. Thomas, W., Mark, B. (2015). *Methods of mathematical modeling: continuous systems and differential equations*. New York: Springer.
2. Sandip, B. (2014). *Mathematical modeling: models, analysis and applications*. Florida: Chapman and Hall, CRC Press.
3. Clive L. D. (2004). *Principles of mathematical modeling* (2nd ed.). Cambridge: Academic Press.
4. Recent research articles.

MATH-7163 **Computer Graphics** **3(3+0)**

Computer Graphics is the illustration field of Computer Science. Its use today spans virtually all scientific fields and is utilized for design, presentation, education and training. Computer Graphics and its derivative, visualization, have become the primary tools by which the flood of information from Computational Science is analyzed. The effective construction of three-dimensional computer-generated illustrations is not only a computer science problem; it is a problem that involves other fields as well. For example, it depends heavily on mathematics for its geometric basis and computational algorithms, and it depends on Physics for its principles of geometric optics (the reflection of light from surfaces determines the displayed color of the surface). It is also the study of a field that provides methods by which illustrations can be generated by others.

Contents

1. Introduction to computer graphics and its applications
2. Overview of raster graphics and transformation pipeline
3. Transformations between different coordinate systems which involve modeling coordinate system
4. Device coordinate system,
5. World coordinate system
6. Normalized coordinate system
7. Display window coordinate system and screen coordinate system
8. Graphics output primitives in drawing of lines
9. polygons
10. Triangles
11. Draw polylines with different line joining methods
12. Attributes of graphics primitives like color
13. Line style and fill style
14. 2D and 3D transformations and viewing
15. Describing and using viewing parameters to change the shape of the object, using viewport to change the ratio of clipping window
16. Differences in viewing and modeling transformations
17. Window clipping by Cohen-Sutherland algorithm

Recommended Texts

1. Donald, H. & Baker, M. P. (2003). *Computer graphics with openGL*. New Jersey: Prentice Hall.
2. Peter, S., & Steve, M. (2015). *Fundamentals of computer graphics*. Florida: A K Peters, CRC Press.

Suggested Readings

1. Richard, S. W., Benjamin, L. (2004), *OpenGL superbible*. Philadelphia: Society for Industrial and Applied Mathematics.
2. Samuel, R. B. (2003). *3D computer graphics, a mathematical introduction with openGL*. Cambridge: Cambridge University Press.
3. Recent research papers

MATH-7164 **Minimal Surfaces** **3(3+0)**

The main objective is to present to students and young researchers how tools from differential geometry and analysis of partial differential equations can be combined to obtain interesting, new results in both fields.Minimal surfaces theory and geometric analysis are very active topics in Brazil. These theories are quite advanced and expect to spur developments in new areas. For example Allen-Cahn equation which models two phases transitions is a counterpart of the minimal surface equation in semilinear elliptic partial differential equations. Since the resolution of the De Gorgi conjecture by the group of non linear PDEs in Chile there has been growing interest in geometric aspects of semilinear elliptic equations. Regularity theory of a minimizer of an elliptic functional is at the origin of the subject, beginning with the work of Modica. De Giorgi's conjecture is related to the Bernstein problem in minimal surface theory.

Contents

1 Regular Surfaces
2 Differentiable functions on surfaces
3 The tangent plane
4 Geometric definition of area
5 Gaussian and mean curvature
6 Curvature in local coordinates
7 Ruled and minimal surfaces
8 Historical survey and introduction to the theory of minimal surfaces
9 Basic minimal surfaces properties
10 Topological and physical properties
11 Stable and unstable minimal surfaces
12 Two dimensional minimal surfaces in three dimensional space
13 Helecoid
14 Catenoid and conoid
15 Harmonic approximation to area
16 Nambu-Goto action
17 Compactness
18 Singularities
19 Topological applications

Recommended Texts

1 Dierkes, U. (2005). *Minimal surfaces* (2nd ed.). New York: Springer.
2 Robert, O. (2002). *A survey of minimal surfaces*. New York: Dover Pub.

Suggested Readings

1 Tobias, H. C., William, P. M. (2011). *A course in minimal surfaces*. Michigan: American Mathematical Society.
2 Ulrich, D., Stefan, H., Anthony, J. T. (2010). *Regularity of minimal surfaces*. New York: Springer-Verlag Berlin.
3 Recent research papers

MATH-7165 **Soliton Wave Theory** **3(3+0)**

In the last thirty years important progress was made in the understanding of properties of certain non-linear differential equations which arise in many different areas of physics, e.g., physics of plasma, solid state physics, biophysics, field theory etc. For these equations, the most prominent of which is the Korteweg-de Vries (KdV) equation, it was possible to find a general method of solution. A common interesting feature is the occurrence of solitons, i.e. stable, non-dissipative and localized configurations behaving in many ways like particles. In the analysis of these equations many interesting mathematical structures were discovered which surprisingly also appear in quantum mechanics and quantum field theory. From a pragmatic point of view these completely soluble non-linear equations are a substantial extension of the 'tool kit' of a physicist which otherwise is mainly restricted to solving linear systems. They also serve as valuable source for intuition about the behavior of non-linear systems.

Contents

1. Introduction to history of solitons.
2. Linear waves. D'Alembert's solution. Dispersion and dispersion relations, wave speed, wavelength, wave number. Linear superposition.
3. Introduction to nonlinear wave phenomena.
4. Types of Travelling Wave Solutions
5. Analysis of the Methods
6. Family of the Korteweg and de Vries equation, its soliton solutions.
7. KdV and mKdV Equations of Higher-orders, its soliton solutions.
8. The Hirota-Satsuma Equations, its soliton solutions.
9. The Benjamin-Bona-Mahony Equation, its soliton solutions.
10. The Medium Equal Width (MEW) Equation
11. The Kawahara and the Modified Kawahara Equations
12. The Kadomtsev-Petviashvili (KP) Equation
13. Boussinesq, Klein-Gordon and Liouville Equations, its soliton solutions.
14. Burgers, Fisher and Related Equations
15. Families of Camassa-Holm and Schrodinger Equations, its soliton solutions.
16. The Generalized Cubic Ginzburg-Landau Equation
17. The nonlinear Schrodinger equation, its soliton solutions.
18. Hirota's method and Bäcklund transformations

Recommended Texts

1. Guo, B., Pang, X.-F., Wang, Y.-F., & Liu, N. (2018). *Solitons*. Berlin: De Gruyter.
2. Shilnikov, L. P., Shilnikov, A. L., Turaev, D. V., Chua, L. O. (2001). *Methods Of Qualitative Theory in Nonlinear Dynamics (Part II)*. World Sci //Singapore, New Jersey, London, Hong Kong.
3. Abdul-Majid Wazwaz. (2009). *Partial Differential Equations and Solitary Waves Theory*. Springer

Suggested Readings

1. Ablowits M. J., Segur, H. (1981). *Solitons and the Inverse Scattering Transforms*. Siam Philadelphia
2. Hirota, R. (2004). *The Direct Method in Soliton Theory*. Cambridge University Press

MATH-7166 **Algorithm Analysis** **3(3+0)**

The aim of the course is to make foundation for the students to understand, design and analyze an algorithm. The main emphasis will be on three areas: algorithms, complexity of algorithms and understanding hard problems interconnecting applications of mathematics to modern Technology. Many problems can be solved by considering them as special cases of general problems while the general problems could be solved by algorithms. Introducing the notion of an algorithmic paradigm, provides a general method for designing algorithms. In crypto-graphic systems, the term key refers to a numerical value used by an algorithm to alter information, making that information secure and visible only to individuals who have the corresponding key to recover the information. Terminating algorithms require stopping criteria and convergence. Many numerical algorithms such as Bisection and Newton-Raphson method have explicit rate of convergence. But still the concept of Turing machine is needed to understand to deal with Halting problems.

Contents

1 Introduction of Algorithms
2 Searching and Sorting techniques
3 Growth of a function
4 Big-O Notation
5 Big-O Estimates for Some Important Functions
6 Growth of Combinations of Functions
7 Big-Omega and Big-Theta Notation
8 Estimating Complexity of algorithm
9 P versus NP problem
10 Floating Point Arithmetic
11 Modular Arithmetic
12 Integer Representations and Algorithms
13 Introduction to cryptography
14 Digital signatures
15 Concept of Turing Machines

Recommended Texts

1 Kenneth H. Rosen. (1999). Discrete Mathematics and its Applications (8th ed.). McGraw Hill, New York
2 Muller J.-M., Brisebarre N., de Dinechin F. (2010) *Handbook of Floating-Point Arithmetic* Cambridge, MA: Birkhäuser Boston

Suggested Readings

1 Bovet, D. P., Crescenzi, P., & Bovet, D. (1994). *Introduction to the Theory of Complexity* (Vol. 7). London: Prentice Hall.
2 Greene, D. H., & Knuth, D. E. (1990). *Mathematics for the Analysis of Algorithms* (Vol. 504). Boston: Birkhäuser.
3 Parberry, I. (2000). Lecture Notes on Algorithm Analysis and Computational Coomplexity. University of North Texas

MATH-7167 **Advanced Mathematical Physics** **3(3-0)**

This course is designed and based on advanced methods of mathematical physics. The course aims to demonstrate the utility and limitations of a variety of powerful computational techniques and to provide a deeper understanding of the mathematics underpinning theoretical physics. This course introduces students to key concepts and techniques in mathematical physics. Topics will be taken from mathematical areas relevant to contemporary research in mathematical physics. The course will be mathematical in nature and does not require a background in physics. Upon successful completion, students will have the knowledge and skills to: 1 Explain the fundamental concepts of a special topic in mathematical physics. 2 Demonstrate accurate and efficient use of specific mathematical physics techniques. 3 Demonstrate capacity for mathematical reasoning through analyzing, proving and explaining concepts from mathematical physics.

Contents

1 Nonlinear ordinary differential equations,
2 Bernoulli's equation, Riccati equation,
3 Lane-Emden equation, Nonlinear Pendulum,
4 Duffing's equation, Pinney's equation,
5 Perturbation theory, Bogoliubov-Krilov method.
6 Linear partial differential equations, classification,
7 Initial and boundary values problems,
8 Fourier analysis, Heat equation, Wave equation, Laplace equation etc.
9 Integral equations, classification, integral transform separable kernels, singular integral equations,
10 Wiener-Hopf equations, Fredholm theory, series solutions. Variational methods,
11 The Euler-Lagrange equations, Solutions to some famous problems,
12 Sturm-Liouville Problem and variational principles,
13 Rayleigh-Ritz Methods for partial differential equations. Matrix algebra,
14 Method of Faddeev, Caley-Hamilton' theorem function of matrices.

Recommended Texts

1 Stephenson, G and Radmore, P.M. (1990). *Advanced Mathematical Methods for Engineering and Science Students*. United Kingdom, Cambridge University Press.
2 Riley, K.F., Hobson, P.M. and Bence, S.J. (2006). *Mathematical Methods for Physics and Engineering*. United Kingdom, Cambridge University Press.
3 Tang, K.T. (2007). *Mathematical Methods for Engineers and Scientists* (Volumes 1,2,3). London: Springer.

Suggested Readings

1 Stone, M. and Goldbart, P. (2009). *Mathematics for Physics*. United Kingdom, Cambridge University Press.
2 Arfken, G.B. and Weber, H.J. (2005). *Mathematical Methods for Physicists*. New York: Academic Press.

MATH-7168 **Advanced Spline Functions** **3(3-0)**

This course work continues to offer a comprehensive treatment of the theory of univariate and tensor-product splines. It will be of interest to researchers and students working in applied analysis, numerical analysis, computer science, and engineering. The material covered provides the reader with the necessary tools for understanding the many applications of splines in such diverse areas as approximation theory, computer-aided geometric design, curve and surface design and fitting, image processing, numerical solution of differential equations, and increasingly in business and the biosciences. This course gives an introduction to the theory and applications of piecewise polynomials represented by B-splines. In the first part the focus is on basic properties and algorithms of B-splines in one variable, and various approximation methods using these. Subsequently the theory and methods are extended to functions in multiple variables/dimensions, such as parametric curves and tensor products. In the last part of the course, the focus is on theoretical properties of B-splines, such as approximation properties and stability.

Contents

1 End conditions for interpolatory spline with unequally spaced knots,
2 Super convergence (Equally-spaced knots),
3 Cubic spline collocation for two point boundary value problems,
4 B-spline representation in terms of divided differences,
5 The B-spline representation of spline functions,
6 Computational considerations: The representation of B-splines (Method based on the recursive definition of divided differences),
7 Method of additional knots,
8 The computation of s(x). The computation of derivatives

Recommended Texts

1 Farin, G. (2002). *Curves and Surfaces for Computer Aided Geometric Design: A Practical Guide*. Academic Press Inc.
2 David, S. (2006). *Curves & surfaces for computer graphics*. New York: Springer Science +Business Media Inc.
3 Bartels, R.H., Bealty, J.C. and Beatty, J.C. (2006). *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*. Morgan Kaufmann Publisher.
4 Faux, I.D. (1979). *Computational Geometry for Design and Manufacture*. Ellis Horwood.

Suggested Readings

1 de Boor, C. (2001). *A Practical Guide to Splines*. UK. Springer Verlag.
2 Schumaker, L.L. (1993). *Spline Functions: Basic Theory*. John Wiley.
3 Wang, R.H. (2005). *Multivariate Spline Functions and Their Applications (Mathematics and its Applications)*. Science Press/ Kluwer Academic Publishers.
4 Bartels, R.H., Bealty, J.C. and Beatty, J.C. (2006). *An Introduction to Spline for use in Computer Graphics and Geometric Modeling*. Morgan Kaufmann Publisher.

MATH-7169 **Theory of Soft Sets and Hybrid Structures** **3 (3+0)**

A soft set is a mathematical tool used to deal with uncertain and vague information. It was introduced by Molodtsov in 1999 as a generalization of classical set theory, which deals with crisp or precise information. In soft set theory, a soft set is defined as a pair (E, F), where E is a non-empty set consisting of parameters (attributes) and F is a mapping from E to the power set of a universal set U. The elements of F are called the soft set parameters, and they represent the degree of membership of the elements of E in a certain set. Soft set theory allows for a flexible definition of membership, as opposed to the crisp membership definition in classical set theory. This flexibility makes soft set theory particularly useful in situations where the membership of elements in a set is uncertain or vague, such as in decision-making, data mining, and pattern recognition. Soft set theory has been applied in various fields, including economics, engineering, and medical diagnosis, among others. It provides a powerful mathematical tool for dealing with uncertain and vague information and has the potential to improve the accuracy of decision-making processes.

Contents:

1. Soft set
2. Properties of Soft Sets
3. Operations of Soft Sets
4. Soft Set Relation and Function
5. Matrix Representation of Soft Sets
6. Hybrid Structures of Soft Sets
7. Hypersoft set
8. Multisoft Set
9. Algebraic Structures based on Soft set and its extensions
10. Topological Structures of Soft set and its extensions

Recommended Books:

1. John, S. J. (2020). *Soft sets: Theory and applications* (Vol. 400). Springer Nature.
2. John, S. J. (Ed.). (2016). *Handbook of Research on Generalized and Hybrid Set Structures and Applications for Soft Computing*. IGI Global.

Suggested Readings:

1. Indurain, E., Fernandez, J., & Bustince, H. (2018). New Trends in Fuzzy Set Theory and Related Items. MDPI-Multidisciplinary Digital Publishing Institute.
2. Molodtsov, D. (1999). Soft set theory—first results. Computers & mathematics with applications, 37(4-5), 19-31.